HW 1-18-23

① Prove that a totally bounded set in any metric space is bounded.

② Consider a convergent sequence in any metric space M, $x_n \to x$. Using the definition of compactness, prove that the set $K = \{x, x_1, x_2, \dots\} \subset M$ is compact.

③ If (M, ρ) is a metric space, define

$$\sigma(x,y) = \min\{1, \rho(x,y)\}, \quad x, y \in M.$$

Show that σ is a metric, and $x_n \to x$ in $(M, \rho) \Longleftrightarrow$
$\Longleftrightarrow x_n \to x$ in (M, σ).