HW 1-20-23

① Let (M, ρ) be an arbitrary metric space. Prove that if a Cauchy sequence $x_n \in M$ has a convergent subsequence, then the whole sequence converges.

② Let M consist of absolutely convergent series of real numbers. That is, $(x_1, x_2, \dots, x_n, \dots) \in M$ means: each $x_n \in \mathbb{R}$, and $\sum_{n=1}^{\infty} |x_n| < \infty$.

(a) Show that if $x = (x_1, x_2, \dots)$ and $y = (y_1, y_2, \dots) \in M$ $\Rightarrow \sum_{n=1}^{\infty} |x_n - y_n|$ converges.

(b) Denoting the sum by $\rho(x, y)$, prove that ρ is a metric on M.

③ Prove that (M, ρ) of Problem 2 is a complete metric space.