HW 1-27-23

① Let $E \subset M$ be arbitrary. Show $E \cup \partial E$ is closed; and if $F \supset E$ is closed, then $F \supset E \cup \partial E$.
(So: $E \cup \partial E$ is the smallest closed set that contains E. It is called the closure of E, denoted $\overline{E}$.)

② Show that $f: M \to P$ is continuous, $Z \subset M$ is closed does not imply $f(Z) \subset P$ is closed. (The solution should involve giving an example. Hint: Let $M = \mathbb{R}^2$, $P = \mathbb{R}$, $f(x,y) = x$. But $Z = ?$)

③ Give an example of a continuous map $g: M \to P$ and a compact $K \subset P$ s.t. $g^{-1}K$ is not compact.

1-30-23

① Let $x_n \in M$ form a Cauchy sequence. If $g: M \to P$ is continuous, does it follow that $g(x_n) \in P$ form a Cauchy sequence? Does it follow if we assume g is uniformly continuous?

② Suppose (M, ρ) is a compact metric space, (P, σ) some metric space, and $f: M \to P$ continuous and bijective (i.e. $\forall y \in P\ \exists$ unique $x \in M$ s.t. $f(x) = y$). Prove that then $f^{-1}: P \to M$ is also continuous.

③ If $\varphi: M \to \mathbb{R}$ and $\psi: M \to \mathbb{R}$ are u.s.c., show that $\varphi + \psi$ is also u.s.c.