

HW 1/9/23

- ① On $M = \mathbb{R}^2$ let $f(x, y) = |x_1 - y_1| + |x_2 - y_2|$, $x = (x_1, x_2)$, $y = (y_1, y_2)$
Show this is a metric, and sketch some balls.
- ② Can you define a metric on \mathbb{R} such that
- (a) Every subset $G \subset \mathbb{R}$ is open?
 - (b) Every sequence in \mathbb{R} converges (in this metric)?
- ③ If in (M, f) we have two convergent sequences $x_n \rightarrow x$ and $y_n \rightarrow y$ ($n \rightarrow \infty$), prove that
- $$\lim_{n \rightarrow \infty} f(x_n, y_n) = f(x, y).$$