HW 2-10-23

(1) Suppose $C \subset M$ has the property that any cover of C by <u>balls</u> has a finite subcover. Prove that C must be compact.

(2) A map $\varphi: M \to M$ is called an isometry if $\rho(\varphi(x), \varphi(y)) = \rho(x, y)$ for all $x, y \in M$. Prove that $I = \{\varphi: M \to M \text{ is an isometry}\} \subset \mathrm{Map}(M, M)$ is compact.

(3) Suppose $f: \mathbb{R} \to \mathbb{R}$ has only finitely many points of discontinuity. Prove that there is a sequence $f_k: \mathbb{R} \to \mathbb{R}$ of continuous functions with $f_k \to f$ $(k \to \infty)$.