HW 2-10-23

(1) Suppose $C \subset M$ has the property that any cover of C by <u>balls</u> has a finite subcover. Prove that C must be compact.

(2) A map $\varphi: M \to M$ is called an isometry if $\rho(\varphi(x), \varphi(y)) = \rho(x,y)$ for all $x, y \in M$. Prove that $I = \{\varphi: M \to M \text{ is an isometry}\} \subset \mathrm{Map}(M, M)$ is compact.

(3) Suppose $f: \mathbb{R} \to \mathbb{R}$ has only finitely many points of discontinuity. Prove that there is a sequence $f_k: \mathbb{R} \to \mathbb{R}$ of continuous functions with $f_k \to f$ $(k \to \infty)$.

2-13-23

(1) Prove that a function $\varphi: M \to P$ is continuous at $a \in M$ $\iff$ for any sequence $x_n \in M$ with $\lim_{n\to\infty} x_n = a$ we have $\varphi(a) = \lim_{n\to\infty} \varphi(x_n)$.

(2) For $f, g: [a,b] \to \mathbb{R}$ show that $T(f \pm g) \leq T(f) + T(g)$; and if $c \in \mathbb{R}$, then $T(cf) = |c| T(f)$.

(3) If $\varphi, \psi \in BV[a,b]$, does it follow that $\varphi\psi \in BV[a,b]$?