

HW 2-17-23

- ① Suppose $A \subset [0, \infty)$, $B \subset (-\infty, 0]$. Prove $m^*(A \cup B) = m^*(A) + m^*(B)$.
- ② Suppose $E \subset \mathbb{R}$ has the following property: for any interval I $m^*(E \cap I) \leq |I|/2$. Prove that E has measure 0.
- ③ Suppose $f: [0, 1] \rightarrow \mathbb{R}$ is differentiable and f' is continuous. Let $C = \{x: f'(x) = 0\}$ denote the set of critical points. Show that $f(C) \subset \mathbb{R}$, the set of critical values, has measure 0.