HW 2-22-23

(1) If $E \subset \mathbb{R}$ is measurable, prove that there is an F_σ set A and a G_δ set B such that $A \subset E \subset B$, and $B \setminus E$, $E \setminus A$ have zero measure.

(2) Suppose $X \subset [0,1]$ is measurable, $m(X) > 1/2$. Prove that the difference set

$$X - X = \{x - y : x, y \in X\}$$

contains some interval $[0, \varepsilon]$ with $\varepsilon > 0$.