

HW 2-24-23

- ① In previous problem #1, show that it may not be possible to choose A to be closed, not just F_σ .
- ② Let $q_n \in (0, 1)$, $n \in \mathbb{N}$. Modify Cantor's original construction of his set so that in the n 'th step we remove the middle q_n 'th of each remaining interval; starting with $[0, 1]$. (So in Cantor's construction $q_n = 1/3 \forall n$.) What measure does the (q_n) -Cantor set thus constructed have? Can it have positive measure for some choice of q_n ?
- ③ Let $r_1, r_2, \dots \in \mathbb{R}$ be a sequence, and $g: \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = \sum_{r_n < x} 2^{-n}$. Show that for any $A \subset \mathbb{R}$
- $$m_g^*(A) = \dots = \sum_{n: r_n \in A} 2^{-n}. \quad \text{What } A \text{ will be measurable?}$$