HW 2-3-23

(1) Let $E \subset M$ and $f(x) = \inf_{y \in E} \rho(x, y)$ (assume $E \neq \emptyset$).
Prove that f is uniformly continuous, and $f^{-1}(0) = E \cup \partial E$.

(2) Suppose $f_n : M \to P$ converge uniformly to a continuous $f : M \to P$. If $x_n \in M$ converge to $x \in M$, prove that $f_n(x_n) \to f(x)$.

(3) Suppose u.s.c. functions $\varphi_k : M \to \mathbb{R}$ converge uniformly to $\varphi : M \to \mathbb{R}$. Prove that φ is also u.s.c.