HW 2-3-23

① Let $E \subset M$ and $f(x) = \inf\limits_{y \in E} \rho(x, y)$ (assume $E \neq \emptyset$).
Prove that f is uniformly continuous, and $f^{-1}(0) = E \cup \partial E$.

② Suppose $f_n : M \to P$ converge uniformly to a continuous $f : M \to P$. If $x_n \in M$ converge to $x \in M$, prove that $f_n(x_n) \to f(x)$.

③ Suppose u.s.c. functions $\varphi_k : M \to \mathbb{R}$ converge uniformly to $\varphi : M \to \mathbb{R}$. Prove that φ is also u.s.c.

2-6-23

① Prove that if the space of maps $Map(M, P)$ with the metric $\tau(f, g) = \sup\limits_{x \in M} \sigma(f(x), g(x))$ is complete, then (P, σ) is complete.

② Suppose $f, g, f_n, g_n : M \to \mathbb{R}$ satisfy: $f_n \to f$ uniformly, $g_n \to g$ uniformly. Does it follow that $f_n + g_n \to f + g$ uniformly? Does it follow that $f_n g_n \to fg$ uniformly?

③ Consider a ~~set~~ $F \subset M$. If $F \cap K$ is closed for every compact $K \subset M$, show that F is closed.