

2-8-23 HW

① Suppose  $(M, \rho)$  is complete,  $M \supset F_1 \supset F_2 \supset \dots \supset F_n \supset \dots$  are closed and non-empty. Assuming  $\lim_{n \rightarrow \infty} \text{diam } F_n = 0$ , prove that  $\bigcap_{n \in \mathbb{N}} F_n \neq \emptyset$ .

② Consider a family  $\mathcal{U} \subset \text{Map}(M, \mathbb{R})$ ,  $\mathcal{U} \neq \emptyset$ . Suppose each  $u \in \mathcal{U}$  is u.s.c. and takes only non-negative values. Show that the function  $v: M \rightarrow \mathbb{R}$  defined by 
$$v(x) = \inf \{ u(x) : u \in \mathcal{U} \}, \quad x \in M,$$
 is also u.s.c.