HW 3-24-23

(1) Evaluate asymptotically $\int_0^\infty e^{-\lambda x} f(x)\,dx$ as $\lambda \to \infty$, in the spirit of an example in class. Here $f \in C[0,\infty)$ is assumed to be bounded.

(2) A measure space $(\Omega, \mathcal{A}, \mu)$ is σ-finite if $\Omega = \bigcup_{n=1}^{\infty} \Omega_n$ with $\mu(\Omega_n) < \infty$, $\forall n \in \mathbb{N}$. Suppose $\psi_1, \psi_2 \geq 0$ are measurable functions such on Ω and

$$\int_E \psi_1 \, d\mu = \int_E \psi_2 \, d\mu \quad \text{for } \forall E \in \mathcal{A}.$$

Prove that $\psi_1 = \psi_2$ a.e.

(3) If $u: \mathbb{R} \to \mathbb{R}$ is Lebesgue measurable, prove that $\exists\, v: \mathbb{R} \to \mathbb{R}$ Borel measurable s.t. $u = v$ a.e.