HW 3-29-23

(1) Suppose $f \in \mathcal{L}^1(\mathbb{R}, \text{Lebesgue})$ and $F(x) = \int_{-\infty}^{x} f$.
Prove that F is differentiable almost everywhere.

(2) Suppose $h \in \mathcal{L}^1(\mathbb{R}, \text{Lebesgue})$, and for every $p, q \in \mathbb{R}$, $p < q$, we have $\int_p^q h \geq 0$. Prove that $\int_E h \geq 0$ for every measurable $E \subset \mathbb{R}$.