

HW 3-3-23

- ① Suppose $\psi \in L(\Omega, \mathcal{A}, \mu)$ is nonnegative. For $E \in \mathcal{A}$ define $\nu(E) = \int \psi \chi_E \in [0, \infty]$. (Here χ_E is the characteristic function of E .) Prove that $(\Omega, \mathcal{A}, \nu)$ is a measure space.
- ② Let \mathcal{A} be a σ -algebra of subsets of Ω and $A_n \in \mathcal{A}$, $n \in \mathbb{N}$. Prove that $\{x \in \Omega : \exists \text{ infinitely many } n \text{ such that } x \in A_n\} \in \mathcal{A}$.
- ③ Suppose $f \in L(\Omega, \mathcal{A}, \mu)$ and $\int f < \infty$. Prove that $f < \infty$ a.e.