HW 3-8-23

(1) Prove that if $f:\mathbb{R}\to\mathbb{R}$ is integrable (with respect to Lebesgue measure) and $t\in\mathbb{R}$, then the function $g(x)=f(x+t)$ is also integrable and $\int_{\mathbb{R}} f=\int_{\mathbb{R}} g$.

(2) In an earlier hw you proved that $f\in L(\Omega,\mathcal{A},\mu)$, $f\geq 0$, $\int f=0$ implies $f=0$ a.e. Show that the following "approximate" version is <u>false</u>:

If $f_k\in L(\Omega,\mathcal{A},\mu)$, $f_k\geq 0$, $k\in\mathbb{N}$, and

$$\lim_{k\to\infty}\int f_k=0 \implies \lim_{k\to\infty} f_k=0 \text{ a.e.}$$