HW 4-7-23

① If $\mu(\Omega)<\infty$, show that $\mathcal{L}^p(\Omega,\mathcal{A},\mu)\supset\mathcal{L}^q(\Omega,\mathcal{A},\mu)$ when $p\le q$.

② Suppose $f:\Omega\to[-\infty,\infty]$. A number $M\in[-\infty,\infty]$ is an essential upper bound of f if $f\le M$ a.e. Prove that the set $\{M : M \text{ is an essential upper bound of } f\}$ has a least element.

③ Construct $f\in\mathcal{L}^1(0,1)$ such that for no $p>1$ is $f\in\mathcal{L}^p(0,1)$.

4-10-23

① If $(\Omega,\mathcal{A},\mu)$ is ($\mathbb{N}$, all subsets, counting measure), the space $L^p(\Omega,\mathcal{A},\mu)$ is denoted ℓ^p. Prove that if $p\le q$, then $\ell^p\subset\ell^q$.

② If $p\ne q$, show that neither of $\mathcal{L}^p(\mathbb{R},\mathcal{M},m)$, $\mathcal{L}^q(\mathbb{R},\mathcal{M},m)$ (Lebesgue spaces) contains the other.

③ Suppose $f_n\in AC[a,b]$ are increasing functions, and $\sum_1^\infty f_n$ converges pointwise. Prove that $\sum_1^\infty f_n\in AC[a,b]$.