

HW 4-12-23

① Suppose $\mu(\Omega) < \infty$ and $f \in L^\infty(\Omega, \mathcal{A}, \mu)$. Prove that $\lim_{p \rightarrow \infty} \|f\|_p = \|f\|_\infty$.

② A family of functions $\mathcal{F} \subset L^p(\Omega, \mathcal{A}, \mu)$ is uniformly p -integrable if $\forall \varepsilon > 0 \exists \delta > 0$ s.t.

$$\int_E |f|^p < \varepsilon \text{ when } \mu(E) < \delta \text{ and } f \in \mathcal{F}.$$

(Here $1 \leq p < \infty$.) Suppose $K \subset L^p(\Omega, \mathcal{A}, \mu)$ is a compact subset. Prove that it is uniformly p -integrable.