HW 4-3-23 continued

(2) Prove that any absolutely continuous function $\varphi : [a,b] \to \mathbb{R}$ is of bounded variation.

(3) If $g : [a,b] \to \mathbb{R}$ is increasing and absolutely continuous, prove that the associated Lebesgue-Stieltjes measure m_g is absolutely continuous w.r.t. to Lebesgue measure m ($m_g \ll m$).

HW 4-5-23

(1) If $f, g \in AC[a,b]$, prove $\int_a^b f'g = fg\Big|_a^b - \int_a^b fg'$.

(2) If $g : [a,b] \to \mathbb{R}$ is increasing and absolutely continuous, and m_g is the associated Lebesgue-Stieltjes measure, prove that for any Borel set $E \subset [a,b]$ $\int_E g' = m_g(E)$.