HW 4-7-23

(1) If $\mu(\Omega) < \infty$, show that $\mathcal{L}^p(\Omega, \mathcal{A}, \mu) \supset \mathcal{L}^q(\Omega, \mathcal{A}, \mu)$ when $p \leq q$.

(2) Suppose $f: \Omega \to [-\infty, \infty]$. A number $M \in [-\infty, \infty]$ is an essential upper bound of f if $f \leq M$ a.e. Prove that the set $\{M : M \text{ is an essential upper bound of } f\}$ has a least element.

(3) Construct $f \in \mathcal{L}^1(0,1)$ such that for no $p > 1$ is $f \in \mathcal{L}^p(0,1)$.