Lecture #22

Today: Review for Midterm #2

- §3.3
  - → Cramer's rule for solutions of $A\vec{x}=\vec{b}$
  - → Formula for A^{-1}: $A^{-1} = \frac{1}{\det A} \cdot \operatorname{adj} A$
  - → Determinants as Area or Volume

- §4.1, 4.2, 4.3, 4.5
  - → Formal notions of a vector space & subspace
  - → Linear Combination, Spanning set
  - → Nul A, Col A, Row A (& their explicit computation)
  - → Kernel & range of linear transformation
  - → Linearly independent/dependent sets
  - → Bases
    - → The Spanning Set theorem
    - → Bases of Nul A, Col A, Row A
  - → Dimension of a vector (sub)space
    - → dim Col A + dim Nul A = # columns

- §5.1, 5.2, 5.3
  - → Eigenvectors & Eigenvalues & Eigenspace
    - → eigenvalues of a triangular matrix
    - → eigenvectors corresponding to distinct eigenvalues are linearly independent
  - → Characteristic Equation, Characteristic polynomial
    - → Find eigenvalues λ from $\det(A-\lambda I)=0$
      - → for each λ found, solve $(A-\lambda I)\vec{v}=\vec{0}$
  - → Similar matrices ($A \rightsquigarrow P^{-1}AP$ - similarity transformation)
  - → Diagonalization ($A = PDP^{-1}$, D-diagonal)
    - → recall the precise algorithm!

Lecture #22

- § 5.4 → Eigenvectors & eigenvalues of linear transformations
  - → The matrix of a linear transformation relative to a basis

$$[T]_{\mathcal{B}} = \left[[T(\vec{b}_1)]_{\mathcal{B}} \dots [T(\vec{b}_n)]_{\mathcal{B}} \right]$$

  - → If $A = PDP^{-1}$ and $\mathcal{B}$ is the basis for $\mathbb{R}^n$ formed by columns of P, then $[T]_{\mathcal{B}} = D$, where $T(\vec{x}) = A\vec{x}$

- Appendix B → Complex numbers $\{a+ib \mid a, b \in \mathbb{R}\}$
  - → division
  - → polar / geometric form $r \cdot e^{i\theta}$
  - → De Moivre's Theorem

- § 5.5 → Complex Eigenvalues
  - → For real matrices, the complex (non-real) eigenvalues come in conjugate pairs
  - → For a real 2×2 matrix A with complex eigenvalue $\lambda = a - b \cdot i$ ($b \neq 0$), one has $A = P \cdot C \cdot P^{-1}$, where $C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

$$P = \begin{pmatrix} \operatorname{Re}\vec{v} & \operatorname{Im}\vec{v} \end{pmatrix}$$

$\vec{v}$ – eigenvector corresp. to eigenvalue λ.