Lecture #12

Today: Review for Midterm #1

- Linear systems
  - → Know: coefficient matrix, augmented matrix
  - → Row reduction (3 types of elementary row operations)
  - → Know: what "consistent" & "inconsistent" stay for.
  - → Reduction to an echelon form and to the reduced echelon form (the latter requires us to move from bottom rows to the upper ones)
  - → Pivot position & pivot column.
  - → Basic vs Free variables
    - Basic: correspond to pivot columns
    - Free: correspond to non-pivot columns
  - → Being able to write down general solution (in case there are infinitely many) via the free variables & in the parametric vector form

- Vectors
  - → Basic operations: addition & scalar multiplication
  - → Linear combination, Span
  - → The way linear system may be viewed as a vector eq-n.
  - → The product of a matrix and a column-vector
  - → The way linear system may be viewed as a matrix eq-n.
  - → $\{A\vec{x}=\vec{b}$ has a solution for any $\vec{b}\}$ $\iff$ $\{$columns of A span $\mathbb{R}^m\}$ $\iff$ $\{A$ has pivot position in each row$\}$. (A: m×n matrix)
  - → Solving homogeneous matrix/vector eq-s in the parametric form
  - → Solving non-homogeneous —//— by finding a single solution and solving the associated homogeneous one.

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- Linear Independence → recall properties of sets containing
  1) $\vec{0}$
  or
  2) having $> m$ elements of $\mathbb{R}^m$

- (Linear) Transformations → Know: domain, codomain, image, range
  - → Matrix transformations
  - → Linear transformations
  - → Linear transf. = Matrix transf.
  - → Recovering the standard matrix of a linear transf.
  - → Know: "onto" and "one-to-one" terminology

    "onto": columns of stand. matrix span $\mathbb{R}^?$

    "one-to-one": columns of stand. matrix are lin. ind.

- Matrices → Addition & Scalar Multiplication
  - → Product (and its interpretation via linear transform.)
  - → Transpose of matrices
  - → Inverse of square matrices
  - → Properties of product, transposition, and inverse

- Inverse Matrices → Explicit f-la for 2x2 matrices
  - → General algorithm: row reduction of $(A \vdots I_n)$
  - → Explicit f-la via the adjugate $\operatorname{adj} A$
  - → Long list of properties equivalent to "A - invertible"

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- **Subspaces**
  - → General definition
  - → Col A, Nul A
  - → Bases of a subspace
  - → Explicit algorithm to find a basis of Col A, Nul A by row-reducing A to the reduced echelon form
  - → Dimension
  - → Coordinates of $\vec{x}$ relative to a basis $\mathcal{B}$: $[\vec{x}]_{\mathcal{B}}$
  - → $\dim \mathrm{Col}\, A =: \mathrm{rank}(A) = \#$ pivot columns

    $\dim \mathrm{Nul}\, A = \#$ non-pivot columns

    $\dim \mathrm{Col}\, A + \dim \mathrm{Nul}\, A = \#$ columns of A
  - → For a k-dimensional subspace $H \subseteq \mathbb{R}^n$, a set of k el-s of H forms a basis of H if just one of the two required conditions holds:
    - either they are lin. ind.

    or

    - they span H.

- **Determinants**
  - → Recursive formula
  - → Cofactor expansion f-la along any row/column
  - → Computation via row reduction to echelon form
  - → "A - invertible" $\Leftrightarrow$ "$\det A \neq 0$"
  - → $\det A = \det A^T$
  - → $\det(AB) = \det A \cdot \det B$
  - → behavior under column reduction.

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Determinants
- → Cramer's rule
- → Formula for A^{-1} via adjugate matrix $\operatorname{adj} A$
- → Geometric meaning of $|\det A|$ for 2×2 and 3×3 matrices
  - → Area / Volume
  - → Ratio of Area/Volume of the image $T(S)$ to that of S for any region S in $\mathbb{R}^2$ or $\mathbb{R}^3$.