# Syllabus for MA 59500
# Lie Algebras
# Fall 2023

## 1. Course Description

This is an introductory course on Lie algebras, used in both mathematics and physics. We will start from Lie groups that provide an actual motivation for the subject, but will quickly diverge to the study of finite-dimensional Lie algebras, with the key emphasis placed on the semisimple Lie algebras (which admit an elegant complete theory). The course is expected to fit a wide variety of students: both graduate as well as advanced undergraduate students.

Lie algebras are just vector spaces equipped with the so-called *Lie bracket*, and they can be often perceived as "infinitesimal" symmetries. While being of independent interest, this subject arises naturally and finds interesting applications in many areas of mathematics and mathematical physics: algebraic combinatorics, differential geometry, topology, number theory, representation theory, partial differential equations, quantum physics, and many more.

**Prerequisites:** A standard course in Linear Algebra and the basic notions from Algebra.

## 2. Course Outline

Major topics to be covered:

- Manifolds and Lie groups
- Lie group actions and homogeneous spaces
- Vector fields, the exponential map, and three fundamental theorems of Lie theory
- The universal enveloping algebra and the Poincaré-Birkhoff-Witt theorem
- Free Lie algebras and the Baker-Campbell-Hausdorff formula
- Nilpotent and solvable Lie algebras, Engel's and Lie's theorems
- Simple, semisimple, reductive Lie algebras and their properties
- The Killing form and Cartan's criteria for solvability and semisimplicity
- The Casimir element, extensions of representations, and Whitehead's theorem
- Cartan subalgebras and the root space decomposition of semisimple Lie algebras
- Strongly regular elements and Chevalley's theorem
- Root systems and properties of their Weyl groups
- Classification of reduced root systems via Dynkin diagrams
- Classification of semisimple Lie algebras and their Chevalley-Serre presentation
- Representation theory of semisimple Lie algebras
- Weyl character and dimension formulas

If time remains: Levi's theorem, Harish-Chandra isomorphism, and BGG resolution.

## 3. Lectures

Time: TTh 3:00–4:15pm
Location: LWSN 1106
Course CRN: #17859
Course Credits: 3

Instructor: Sasha Tsymbaliuk
Email: otsymbal@purdue.edu (emails will be responded within 24h Mon-Fri)
Office hours: TTh 1:00–2:30pm (Math Building 620)

Instructional Modality: Face-to-Face
Homework Assignments and Lecture Notes: will be posted on Brightspace

## 4. References

There is no required textbook. The recommended references are:

(1) Book "An Introduction to Lie Groups and Lie Algebras" by Alexander Kirillov, Jr. Cambridge Studies in Advanced Mathematics (2017), ISBN:978-1316614105.
(2) Book "Introduction to Lie Algebras and Representation Theory" by James Humphreys Springer (1978), ISBN:0-387-90053-5.

Some other relevant literature is:

(3) Book "Introduction to Lie Algebras" by Karin Erdmann and Mark Wildon Springer Undergraduate Mathematics Series (2006), ISBN:978-1-84628-040-5.
(4) Book "Lie algebras and Lie groups" by Jean-Pierre Serre Lecture Notes in Mathematics (2006), ISBN:978-3-540-55008-2.
(5) Book "Lie algebras of finite and affine type" by Roger Carter Cambridge University Press (2005), ISBN:978-0-521-85138-1.
(6) Book "Representation Theory. A first course" by William Fulton and Joe Harris Springer (1999), ISBN:0-387-97527-6.

## 5. Requirements

If you are taking this course for credit, it will be required to solve weekly homework assignments. The assignments will be posted each Thursday and due the following Thursday, and will consist of 6-9 problems of various difficulty. There will be no final exam.

To get $A-$, a graduate student is expected to solve most of the simpler problems, while to get grades $A, A+$ one should also solve some of the harder/more technical problems (marked by an asterisk $^\star$). The University's policy is that students who get at least 97% of the total points in this course are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77% a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-. For each of these grades, the lower percentage will most probably suffice (as noted above).

For undergraduate students, there is an additional requirement to turn in a short (3-6 pages) expositional paper on one of the topics covered in the class (or a closely connected one). The choice of a topic shall be discussed with each student individually later in the term, and the final paper shall be due at the beginning of the Final Exam's week.

## 6. Resources and Collaboration

While solutions are often available online, please make every effort to solve problems yourselves (in case you had to look up for hints or solutions, please cite the source accordingly).

Working in groups is fine, but each person should write up their solutions independently.

## 7. Attendance

Attendance is expected overall. Frequent absences may affect the grade.

Do not come to class if you are feeling ill, but do email me with the subject line: Absence (no need to describe symptoms, but please notify you are feeling ill and cannot come to class).

## 8. Academic Adjustements for Students with Disabilities

Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are welcome to let me know so that we can discuss options. You are also encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247.

If you have been certified by the Disability Resource Center (DRC) as eligible for accommodations, you should contact your instructor to discuss your accommodations as soon as possible. Here are instructions for sending your Course Accessibility Letter to your instructor: https://www.purdue.edu/drc/students/course-accessibility-letter.php

## 9. Emergency Preparation

In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors via email.

A link to Purdue's Information on Emergency Preparation and Planning is located on our Brightspace under "University Policies and Statements". This website covers topics such as Severe Weather Guidance, Emergency Plans, and a place to sign up for the Emergency Warning Notification System. I encourage you to download and review the Emergency Preparedness for Classrooms document (PDF) or (Word). The first day of class, I will review the Emergency Preparedness plan for our specific classroom, following Purdue's required Emergency Preparedness Briefing. Please make note of items like:

– The location to where we will proceed after evacuating the building if we hear a fire alarm.
– The location of our Shelter in Place in the event of a tornado warning.
– The location of our Shelter in Place in the event of an active threat such as a shooting.