HOMEWORK 1 (DUE FEBRUARY 2)

Part 1: Pick and write up solutions for any 3 problems among the ones below.

1. Exercise 1 in III.8 of Kassel's textbook, pages 66–67.
2. Exercises 2&5(a,b,c) in III.8 of Kassel's textbook, pages 67–68.
3. Exercise 6 in III.8 of Kassel's textbook, page 68.
4. Exercises 8&9 in III.8 of Kassel's textbook, page 69.
5. Exercise 10 in III.8 of Kassel's textbook, page 69.
6. Exercise 5 in IV.9 of Kassel's textbook, page 89.

Part 2: Pick and write up solutions for any 1 problem among the ones below.

7. Exercise 6 in IV.9 of Kassel's textbook, page 90.
8. Exercise 7 in IV.9 of Kassel's textbook, page 90.

Part 3 (optional extra problem*): Prove the *structure theorem for bimodules* below.

Theorem (the structure theorem for bimodules, III.9 of Kassel's textbook, bottom of page 71): *Let H be a Hopf algebra and M be an H-bimodule. Consider a subspace $N \subset M$ defined by*

$$N := \left\{ m \in M \,\middle|\, \Delta_M(m) = 1 \otimes m \right\}.$$

Then, the multiplication map $\mu_M \colon H \otimes N \to M$ is an isomorphism of H-bimodules.

The suggested scheme of proof consists of the following four steps:

(a) Define a linear map $\gamma \colon M \to M$ as the following composition

$$M \xrightarrow{\Delta_M} H \otimes M \xrightarrow{S \otimes \mathrm{Id}_M} H \otimes M \xrightarrow{\mu_M} M.$$

Prove that $\gamma(M) \subset N$.

Next, we consider two linear maps $\alpha \colon H \otimes N \to M$ and $\beta \colon M \to H \otimes N$ defined by

$$\alpha(h \otimes n) = \mu_M(h \otimes n) \quad \text{and} \quad \beta(m) = (\mathrm{Id}_H \otimes \gamma)(\Delta_M(m)).$$

(b) Verify $\alpha \circ \beta = \mathrm{Id}_M$.

(c) Verify $\beta \circ \alpha = \mathrm{Id}_{H \otimes N}$.

(d) Verify that α is a morphism of H-comodules.