Syllabus for MA 59500
Quantum Groups and Applications
Spring 2024

1. Course Description

This is an introductory course to (finite) quantum groups, the subject ubiquitous in various branches of modern mathematics and mathematical physics. The course is expected to fit a wide variety of math and physics students: both graduate as well as advanced undergraduate.

The theory of quantum groups originates from the math-physics work of Faddeev's school in mid 80s in the study of quantum inverse scattering method. The theory was soon generalized in the papers of Drinfeld and Jimbo through the formalism of Hopf algebras created by algebraic topologists in the middle of the 20th century. The basics of this theory were outlined in Drinfeld's 1986 ICM talk, which still remains an excellent reference on the subject.

Prerequisites: A basic knowledge of simple Lie algebras (e.g. Fall 2023 course MA59500L). Familiarity with basic notions of category theory will be useful for the second part of the course.

2. Course Outline

In the first part of the course, we shall cover:

- Coalgebras, bialgebras, modules and comodules
- Hopf algebras, examples of $U(\mathfrak{sl}_n)$ and $\mathbb{C}[SL(n)]$, and the pairing between them
- Quantum plane, quantum $SL_q(2)$, quantum $U_q(\mathfrak{sl}_2)$
- Quantum groups $U_q(\mathfrak{g})$ and their Hopf algebra structure
- Finite dimensional representations of $U_q(\mathfrak{g})$ for $q \neq \sqrt{1}$
- Center of $U_q(\mathfrak{g})$ and non-degenerate pairings
- Lusztig's braid group action, root vectors, PBW bases, and factorization of R-matrices
- Braided and cobraided bialgebras, and Faddeev-Reshetikhin-Takhtajan construction

In the second part of the course, we shall have a basic introduction to:

- Tensor categories, tensor functors, and braided tensor categories
- Knots, links, link diagrams, Reidemeister moves, and Jones-Conway polynomial
- Tangles, tangle category, braids, braid category, and braid group

3. Lectures

Time: MWF 1:30–2:20pm
Location: SCHM 317
Course CRN: #19741
Course Credits: 3

Instructor: Sasha Tsymbaliuk
Email: otsymbal@purdue.edu (emails will be responded within 24h Mon-Fri)
Office hours: MF 12:00–1:30pm (Math Building 620)
Instructional Modality: Face-to-Face (or Online if I need to quarantine or travel)

4. References

The material will be mostly based on two classic books:

(1) Book "Lectures on quantum groups" by Jens Jantzen (1996)
Graduate Studies in Mathematics Vol. 6, ISBN: 978-1-4704-2067-3
(2) Book "Quantum groups" by Christian Kassel (1995)
Graduate Texts in Mathematics Vol. 155, ISBN: 978-0-387-94370-1

Free versions of both books can be found online. Additionally, Purdue Library has provided a link to the Kassel's book which is now deposited in the Brightspace→Content→Literature.

5. Requirements

If you are taking this course for credit, it will be required to solve biweekly homework assignments. The assignments will be posted each other Friday and due two Fridays afterwards, and will consist of 6–10 problems of various difficulty. There will be no exams in this course.

To get $A-$, a graduate student is expected to solve most of the simpler problems, while to get grades $A, A+$ one should also solve some of the harder (marked by an asterisk *) or more technical problems. The University's policy is that students who get at least 97% of the total points in this course are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77% a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-. For each of these grades, the lower percentage will most probably suffice (as noted above).

6. Resources and Collaboration

While solutions are often available online, please make every effort to solve problems yourselves (in case you had to look up for hints or solutions, please cite the source accordingly).

Working in groups is fine, but each person should write up their solutions independently.

7. Attendance

Attendance is expected overall. Frequent absences may affect the grade.

Do not come to class if you are feeling ill, but do email me with the subject line: Absence (no need to describe symptoms, but please notify you are feeling ill and cannot come to class).

8. Academic Adjustements for Students with Disabilities

Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are welcome to let me know so that we can discuss options. You are also encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247.

If you have been certified by the Disability Resource Center (DRC) as eligible for accommodations, you should contact your instructor to discuss your accommodations as soon as possible. Here are instructions for sending your Course Accessibility Letter to your instructor: https://www.purdue.edu/drc/students/course-accessibility-letter.php

9. Emergency Preparation

In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors via email.

A link to Purdue's Information on Emergency Preparation and Planning is located on our Brightspace under "University Policies and Statements". This website covers topics such as Severe Weather Guidance, Emergency Plans, and a place to sign up for the Emergency Warning Notification System. I encourage you to download and review the Emergency Preparedness for Classrooms document (PDF) or (Word). The first day of class, I will review the Emergency Preparedness plan for our specific classroom, following Purdue's required Emergency Preparedness Briefing. Please make note of items like:

– The location to where we will proceed after evacuating the building if we hear a fire alarm.
– The location of our Shelter in Place in the event of a tornado warning.
– The location of our Shelter in Place in the event of an active threat such as a shooting.