Syllabus for MA 59800
Infinite dimensional Lie Algebras and Applications
Spring 2021

1. About the Course

This course will be a detailed introduction, with proofs, into the structure and representation theory of some of the most important infinite dimensional Lie algebras: Heisenberg algebras, Kac-Moody algebras, and Virasoro algebra.

Major topics to be covered:

- Heisenberg algebra, Virasoro algebra, and affine $\widehat{\mathfrak{g}}$ as universal central extensions
- Representations of Heisenberg algebra, Virasoro algebra, affine $\widehat{\mathfrak{sl}}_n$ via Lie algebras $\mathfrak{gl}_\infty, \mathfrak{a}_\infty$, and application to integrable systems
- Boson-fermion correspondence: vertex operator construction and Schur polynomials
- Feigin-Fuchs-Kac determinant formula for Virasoro and the region of unitarity
- The Sugawara construction and the Goddard-Kent-Olive construction
- Structure and representation theory of Kac-Moody algebras
- The Weyl-Kac character formula and the Kac-Macdonald identities
- Shapovalov-Jantzen-Kac-Kazhdan determinant formula for Kac-Moody algebras

2. Lectures

Mode: Synchronous Online via Zoom (recorded lectures will be uploaded to the Brightspace)
Time: TTh 1:30–2:45pm

Instructor: Sasha Tsymbaliuk
Email: otsymbal@purdue.edu
Office hours: TTh 3:00–4:00pm via Zoom

3. References

The material of this course is based on:

(1) Book "*Bombay lectures on highest weight representations of infinite dimensional Lie algebras*" by V. Kac and A. Raina, 2nd edition, 2013.
(2) Expository paper "*Representations of contragredient Lie algebras and the Kac-Macdonald identities*" by B. Feigin and A. Zelevinsky, 1971 (to be distributed in the class).
(3) Book "*Infinite dimensional Lie algebras*" by V. Kac, 1983.

4. Requirements

To pass the course it will be required to solve homework assignments, which will be assigned every Thursday and due the following Thursday. Besides the mandatory problems, the homework assignments will contain optional harder problems, marked by an asterisk *.

5. Academic Adjustements for Students with Disabilities

Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247.

In this mathematics course accommodations are managed between the instructor, the student and DRC Testing Center.

If you have been certified by the Disability Resource Center (DRC) as eligible for accommodations, you should contact your instructor to discuss your accommodations as soon as possible. Here are instructions for sending your Course Accessibility Letter to your instructor: https://www.purdue.edu/drc/students/course-accessibility-letter.php