

• A few organizational remarks:

- Are the scanned notes legible to read?

- Have a mini-election: ask if hwks should be by Monday or Wednesday

→ Outcome to be announced next time.

- Tell that each homework will consist of  $\sim 6$  problems, out of which only  $\sim 3$  will be graded, but you don't know which, so you are expected to do all of them.

- Ask if the current schedule of office hours works for majority

OH RO1: Mon, 13<sup>00</sup>-14<sup>00</sup>

OH RO2: Mon, 12<sup>00</sup>-13<sup>00</sup> + MLC facilities.

OH RO3: Th, 15<sup>00</sup>-16<sup>00</sup>

OH Lec: Mon, Wed 10<sup>00</sup>-11<sup>00</sup>

- ! You should always write your recitation numbers on all submitted homeworks and exams

(this is required to sort you out into your recitation groups)

- The first hwk is due next WED, 9<sup>53</sup>. NO LATE SUBMISSIONS!

• Terminology from last time:

- When we draw a table e.g.

	8	3
1 <sup>st</sup>	A	B
2 <sup>nd</sup>	B	A

it means that there were

$\underbrace{8+3}_{11}$  voters and their ballots

can be split in two groups: • 8 ballots with A as top priority

• 3 ballots with B as top priority.

Definition: This table is called preference schedule for an election.

The same data can be rewritten in the following way

	8	3
A	1 <sup>st</sup>	2 <sup>nd</sup>
B	2 <sup>nd</sup>	1 <sup>st</sup>

In such a case we call this table alternative preference schedule

• Last time we discussed the "Plurality Method" - the simplest method. At the end we discussed a few drawbacks of it: remind those...

N.B. #3: the last of the three drawbacks mentioned last time is that a candidate may be preferred by the voters over all other candidates and yet not win!

Today we are going to learn two more voting methods [see §1.3, 1.4] in your textbook

• The "Borda count method"

This method has a simple idea: give points not only to the 1<sup>st</sup> candidate, but rate all candidates.

Usually, in an election with  $N$  candidates, they give  $N$  points for a first place vote,  $N-1$  points for a second place, ..., 1 point for the last place. After that, they compute how much points each candidate gets and the candidate with the highest total is the winner.

Rem: If the question is to rank all candidates, then we first choose the candidate with the highest score, second we choose the candidate with the second-most points, etc.

[Compare to Problem 1.3.24 from hwk]

\*Let us illustrate how it works on the example from last time:

Number of voters	15	10	11	4
1 <sup>st</sup>	A (60)	C (40)	D (44)	B (16)
2 <sup>nd</sup>	B (45)	B (30)	B (33)	C (12)
3 <sup>d</sup>	C (30)	D (20)	A (22)	D (8)
4 <sup>th</sup>	D (15)	A (10)	C (11)	A (4)

Summing up, we see that in total:

A gets  $60 + 10 + 22 + 4 = 96$  points ← 2<sup>nd</sup>

B gets  $45 + 30 + 33 + 16 = 124$  points ← 1<sup>st</sup>

C gets  $30 + 40 + 11 + 12 = 93$  points ← 3<sup>d</sup>

D gets  $15 + 20 + 44 + 8 = 87$  points ← 4<sup>th</sup>

Note: When using the plurality method, the ranking would be 1<sup>st</sup>: A, 2<sup>nd</sup>: D, 3<sup>d</sup>: C, 4<sup>th</sup>: B.

Remarks: The Borda count method admits a few variations, which we will only mention in the moment and we will return to them on Friday if time permits

(1) Sometimes they use the Borda count with truncated preference ballots: each voter chooses top 3 candidates. In this case, they use <sup>preference</sup> ballots of the 2<sup>nd</sup> kind.

Computation of points is even easier (explain on the previous example)

(2) In some of the elections, they believe the gap between the 1<sup>st</sup> and 2<sup>nd</sup> preferences should be bigger than the gap between 2<sup>nd</sup> and 3<sup>rd</sup> preferences. The only difference is that we multiply by 1, 2, 3, ...,  $N-1$ ,  $> N$  some bigger number.

In hwks and midterms we will be dealing only with the standard Borda count, so you should not worry about these 2 remarks.

(but it is useful to get a general idea that we count points through an algorithm where each preference has its own coefficient)

## • The "Plurality - with - Elimination Method"

Another voting method for today is our Monday's Plurality Method with a new modification, which helps to resolve the 1<sup>st</sup> drawback recalled today.

Demand: the winner needs a majority of the votes to get elected  
 $\geq 50\%$

If there are only two candidates, then the plurality method can be applied on the nose. However, if there are more candidates, there are high chances that everyone will get  $< 50\%$

Q: How should we fix it?

Guess 1: As it is implemented in many TV shows, like X-factor or American Idol, after a certain number of candidates is chosen, they do the following:

- have elections (using plurality method and single-choice ballots)  
 ↳ eliminate the candidate with the fewest votes

- after that, have another round of elections  
 ↳ eliminate the candidate with the fewest votes.

⋮  
 - repeat until only one candidate is remained.

Outcome: Proclaim this candidate the winner.

↑  
"Runoff Method"

However, in many local elections such procedures would be very long and would request a lot of money, since each round of elections is quite expensive.

Therefore, it is common to use the following modification (see next page)

The following modification of the "Runoff method" is called:  
plurality-with-elimination or instant runoff method.

Idea: Have only 1 election with a preference ballot

↳ Eliminate the candidate with the fewest votes (based on the count of first-place votes)

↳ Cross out this name on all ballots and transfer all his votes to the next candidates etc.

↳ Again re-compute the first-place votes for all candidates (still remaining)

↳ Eliminate the one with the fewest votes

⋮  
 proceed until at some point we have a candidate who has  $>50\%$  votes.

Outcome: This candidate is the winner!

\* Let us consider again an example from last time

Number of Voters	15	10	11	4
1 <sup>st</sup>	A	C	D	B
2 <sup>nd</sup>	B	B	B	C
3 <sup>rd</sup>	C	D	A	D
4 <sup>th</sup>	D	A	C	A

Round 1

A	B	C	D
15	4	10	11

B has the fewest votes  
 $\Rightarrow$  eliminate B

Round 2

A	C	D
15	14	11

Here 14 votes for C consist of initial 10 and the 4 votes which got transferred from B.

D has the fewest votes  $\Rightarrow$  eliminate D

Round 3

A	C
26	14

Here  $26 = 15 + 11$   
 transferred from D

Hence, A has the majority of votes  $\Rightarrow$  A is the winner

The complete ranking is: 1<sup>st</sup> (A), 2<sup>nd</sup> (C), 3<sup>rd</sup> (D), 4<sup>th</sup> (B)