Lecture #12 09/26/2016

- Today we are going to finish the topic "Mathematics of Getting Around".
  - On Wedn, we will have a review session for Midterm 1:

$19^{00}-20^{30}$, Sept 28, Heavy Engineering 201

  - If you have any concerns on your grades - come to review session and bring your graded hwks.

- Ask if there are any q-s on the last homework.

Comments:

- In problem # 5.2.8 you are asked to find degree, adjacent vertices, adjacent edges given a graph without drawing it.
  [Consider a simple example in class]
  You can draw it somewhere, but don't tell us about this.
- Ask what are the circuits of degrees 1 and 2 in general?
- In problems where you are asked to find an Euler circuit / path, you are supposed to check first # odd vertices. If it is >2, then you refer to Thms 1-2 from last time and say "No Euler path / circuit exists"

  However, if Thms tell you it exists, you are supposed to provide just one of all possible. Use the Fleury's Algorithm from last time.
- I suggest, you have 2 pictures of the same graph when applying the Fleury's Algorithm, and erase edges of one of them as you make every step.

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- Remark on isolated points
- Remark on the degree' counts coming from loops
- Remark on Thm 2 from last time

} All posted on Blackboard

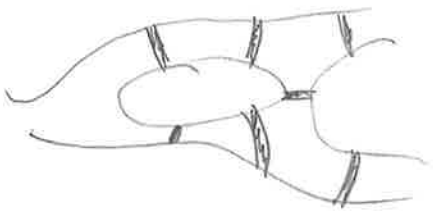
Comment 1: When looking for pathes / circuits, we are not allowed to use the same edge twice, BUT in case there are multiple edges between two vertices, we may use each of them once.

Comment 2: The material from last lecture allows us to address the 1st q-n from last Wednesday: if it is possible to find a route for a UPS driver / mail carrier / security guard such that they fulfill their duties without crossing any street more then needed.

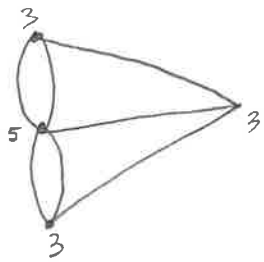
However, it does not provide any optimizational process in case no such route exists. In other words, even if there is no such route, we still would like to find the one which has the least distance "overwalked".

But before we switch to this last q-n, let me give an interesting example of real-life problem, from which Euler elaborated all his ideas in this topic.

Königsberg Bridges



Associated Graph →



Thm 2 ⇒ NO EULER PATH

⇓

NO.

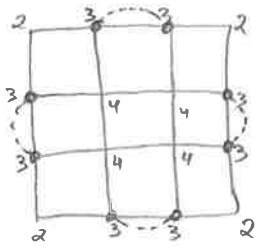
Q: Is it possible to cross each bridge once?

• Eulerizing Graphs

Our goal is to find the shortest route crossing each edge at least once. In other words, we want to minimize the number of "deadhead travel" (in our examples, this is # times we recrossed edges).

Let us illustrate our approach by a simple example.

Ex 1: Find the optimal route that covers all the edges of the graph below and ends back at the starting point.



* First, we find all the odd vertices (there are 8 of them).
* Next, we add duplicates of the edges as shown on the picture by dotted lines.

After that all vertices become even ⇒ there is an Euler circuit.

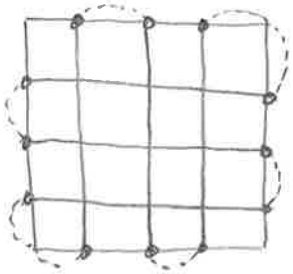
Rmk: We "added" 4 edges, i.e. we recrossed 4 extra times.

Q: (1) Explain why this is optimal.

(2) What will be the answer if we don't have to end at the starting point?

this modification is called semi-Eulerization!

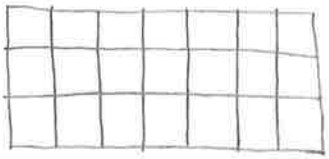
Ex 2: Same question for



Answer: Minimal length is 48 if we need to end at the starting point
46 if we can start and end at different points

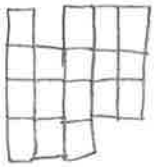
Ex 3: In the problem of Königsberg bridges, what is the least number of bridges we need to recross if
(a) we want to end up at the starting point
(b) we can end up anywhere.

Ex 4: Same question for



[Textbook, Problem 5.4.43]

Ex 5: Same question for



[Textbook, Problem 5.4.46]

Summary:
We learnt the basics of the graph theory and how it can be applied to the real-life problems of optimization.
The key results you should remember:
- Euler's Circuit / Path Theorems, which give a criteria for determining if the Euler Circuit or Path exists.
- Fleury's Algorithm to find an Euler circuit / path if we know its existence.
- Eulerizing and Semi-Eulerizing graphs to find the optimal route in case no Euler Circuit / Path exists