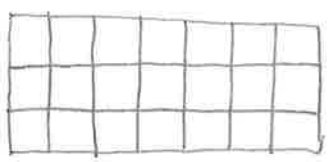
Lecture #13 09/28/2016

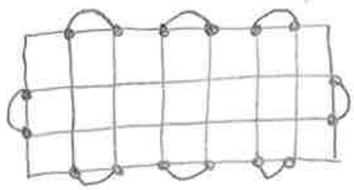
- Discuss the Eularization and semiEularization by working out a few examples.
  - Eularization: want to add double edges so that an Euler circuit exists, i.e. NO odd vertices
  - semiEularization: want to add double edges so that an Euler path exists, i.e. no more than 2 odd vertices

Ex1: Consider an example of Köninsberg bridges (page 2 of Lecture #12)

Ex2: Find an optimal Eularization for the graph below:

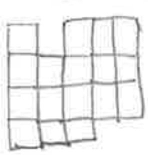


Answer:

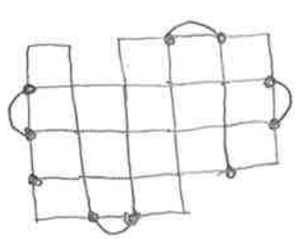


There are 16 odd vertices originally. Want none at the end. Since each new edge changes degrees of exactly two vertices, we need to add at least $\frac{16}{2}=8$ edges, and the picture to the left provides such an example. Hence, it is an optimal Eularization ∎

Ex3: Find an optimal semi-Eularization of the graph below:



Answer:



There are 10 odd vertices originally, while we need at most 2 at the end. Hence, same argument as in Ex2 shows that $\#(\text{new edges}) \geq \frac{1}{2}(10-2)=4$, while the picture to the left gives such an example ∎

NB. If asked to find an optimal Eularization of that graph, it would be much harder to prove "optimal". I expect you always to compare to $\frac{1}{2}\cdot\#(\text{odd vertices to be changed to even})$ and say why you need more than that number if it is a case.

- Consider a few examples on the Fleury's Algorithm