• Ask if there are any q-s on the previous lecture and/or the first 4 problems from new hwk.

• Today: New topic "Population Growth Models" (Section 9 of your Textbook)

Here population originally referred to human populations, but in what we are going to discuss it may be applied to populations of animals, insects, bacteria,... or actually any quantity which changes discretely.

To make sense of the Population Sequences, we need to recall what are the Sequences.

• Sequences

   * infinite ordered set of numbers (each number is called a term)

   * sequence notation: usually we denote sequences as

        $A_1, A_2, A_3, A_4, ---$ or   say $B_0, B_1, B_2, ---$ etc.

   * ways to describe a sequence

      – find the pattern, e.g. if you are given $1, 2, 4, 8, 16, ...$ you can guess that this is just a sequence of powers of 2, and the next term is 32

      – describe sequences by f-las, e.g. $A_n = 2^{n-1}$ – gives above seq. More interestingly, we can also consider $F_n = \left[\left(\frac{1+\sqrt{5}}{2} \right)^n / \sqrt{5} \right]$, so that $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8, ...$ This is an important seq!

      – describe sequences by recursive f-la, e.g. $F_1 = 1, F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ recovers the aforementioned sequence, while $A_1 = 1, A_{n+1} = 2 \cdot A_n$ recovers the former sequence.

|| The sequence $\{F_n\}$ from above is called the Fibonacci sequence.

Rmk: Recursive f-la can be simpler, but you need to compute all preceding terms

                                                                                                    (1)

Lecture #19

Back to our discussion, we will be working with the population sequences, which are used to describe size of population as it changes over discrete time.

- We always have initial population, which we think of as a population at $t=0$. This is the first term of our population sequence.
- As time goes by, the population changes with discrete set of transitions
- The population after the 1st transition is called the first generation
- The population after the 2nd transition is called the second generation

etc...

To fit this terminology, we will use the following notation

$$P_0, \quad P_1, \quad P_2, \quad P_3, \ldots \ldots ,$$

where $P_0 =$ size of initial population, $P_1 =$ size of 1st generation,...

- <u>Visualizing population sequence</u>
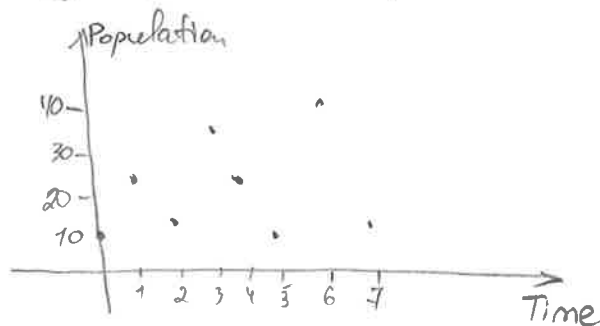
It is convenient to visualize by scatter plot (as on the picture) or line graph (if you connect pts).

- Discuss Example 9.5 on Fibonacci's Rabbits.

- In real life, we rarely have strict mathematical models implemented precisely into actual problems. However, on the big scale those models give a very good approximation on the big scale. The idea is that mathematical model should keep track only of the most influential factors, ignoring some minor changes.

- The first population growth model we are gonna learn is "**Linear Growth Model**"

Def: A population grows according to a <u>linear growth model</u> if in each generation the population changes by a constant amount.

We say that population growth <u>linearly</u>

The corresponding population sequence is an <u>arithmetic sequence</u>.

Example 1: $P_0 = 2000$, $P_1 = 2100$, $P_2 = 2200$, ... $P_{n+1} = P_n + 100$.

In such q-s it is very easy to write a f-la for P_n: $P_n = 2000 + 100n$

Example 2: $P_0 = 2000$, $P_1 = 1900$, $P_2 = 1800$, ____ $P_{n+1} = P_n - 100$.

So: $P_n = 2000 - 100n$

Rmk: While the population in the Ex1 has positive growth, the one in Ex2 has negative growth.

Common Q-s: (1) In Ex1, how long would it take to get a population of 10000? (A: 80)

(2) In Ex2, how long would it take to reach a zero population? (A: 20)

Note: If we draw a scatter plot or linear graph for linear growth models, we would get one of the following

Positive Growth   Zero growth   Negative Growth

③