

Lecture #20

Ex2 (Exercise 19 from your textbook): Consider a population that grows linearly following the recursive formula  $P_N = P_{N-1} + 125$ , with the initial population  $P_0 = 80$ .

(a) Find  $P_1, P_2, P_3$

► (b)  $P_N = 80 + 125N$

(b) Give an explicit formula for  $P_N$

(c)  $P_{100} = 80 + 125 \cdot 100 = 12580$

(c) Find  $P_{100}$

(Answer: (a)  $P_1 = 205, P_2 = 330, P_3 = 455$ ; (b)  $P_N = 80 + 125N$ , (c)  $P_{100} = 12580$ )

(Exercise 3.2.33)

Ex3: Consider a population that grows according to a linear growth model. The initial population is  $P_0 = 23$ , and the common difference is  $d = 7$ .

(a) Find  $P_0 + P_1 + \dots + P_{999}$

(b) Find  $P_{100} + \dots + P_{999}$

► (a)  $P_0 = 23, P_{999} = P_0 + 999 \cdot d = 23 + 999 \cdot 7 = 7016 \Rightarrow P_0 + \dots + P_{999} = \frac{1000 \cdot (P_0 + P_{999})}{2} = 3,519,500$

(b)  $P_{100} = P_0 + 100 \cdot d = 723 \Rightarrow P_{100} + \dots + P_{999} = \frac{900 \cdot (P_{100} + P_{999})}{2} = 3,482,550$ .

ASK if there are any q-s about these 2 examples.

- Next population growth model : "Exponential Growth Model".

We start our discussion from the following notion:

Def: The growth rate of a population as it changes from an initial value  $X$  to a new value  $Y$  is given by the ratio

$$\tau = \frac{Y-X}{X} \quad \rightarrow \text{converted into \% by multiplying by } 100\%.$$

Terminology: In the above setup, we call  $X$  the baseline,  $Y$  - the end-value.

Ex 4: Fill in the following table

(if there are q-s, give a few more examples)

| Baseline ( $X$ ) | End-Value ( $Y$ ) | Growth rate % ( $\tau$ ) |
|------------------|-------------------|--------------------------|
| 10               | 30                |                          |
| 20               |                   | 50%                      |
|                  | 60                | 100%                     |