Lecture #22 10/21/2016

· Last time: "Logistic Growth Model"

Key equation: $\boxed{p_{n+1} = r \cdot (1-p_n) \cdot p_n}$, where $p_n = \frac{P_n}{C}$ ← population of $n^{th}$ generation; $C$ ← carrying capacity

This recursive f-la allows us to compute all terms once the values of $r$ (growth parameter) and $p_0$ (seed) are given.

Ex1 (Example 9.18): Compute $p_n$ for all $n$ given $p_0 = 0.2$, $r = 2.5$.

$p_1 = r \cdot p_0 \cdot (1-p_0) = 2.5 \cdot 0.2 \cdot 0.8 = 0.4$

$p_2 = r \cdot p_1 \cdot (1-p_1) = 2.5 \cdot 0.4 \cdot 0.6 = 0.6$

$p_3 = r \cdot p_2 \cdot (1-p_2) = 2.5 \cdot 0.6 \cdot 0.4 = 0.6.$

We see that $p_3 = p_2$. Since $p_4$ is expressed via $p_3$ in the same way as $p_3$ via $p_2$, we also get $p_4 = p_3 = p_2$ etc.

So: $p_0 = 0.2$, $p_1 = 0.4$, $p_2 = p_3 = p_4 = \ldots = 0.6$. ■

(this is the case of stable equilibrium)

Ex2 (Exercise 9.4.59): Given a logistic growth model with $r = 2.8$, $p_0 = 0.15$, find the values of $p_1$ through $p_{10}$.

$p_1 \approx 0.3570$

$p_2 \approx 0.6427$

$p_3 \approx 0.6429$

$p_4 \approx 0.6428$

$p_5 \approx 0.6429$

$p_6 \approx 0.6428$

$p_7 \approx 0.6429$

$p_8 \approx 0.6428$

$p_9 \approx 0.6429$

$p_{10} \approx 0.6428$

N.B.: Actually as $n$ grows, the population stabilizes to $\frac{9}{14}$ of the carrying capacity. ■

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- Today: Financial Mathematics

## §10.1 Percentages

As already discussed last time, any coefficient can be transfered to percentage by multiplying by 100%. (This is useful in these situations when you have to compare different coeff, e.g. $\frac{18}{25}$ vs $\frac{16}{20}$)

Ex1: If you got 18 pts out of 25 on your hwk1, it means you got 72%, while getting 16 pts out of 20 on hwk 2 means you got 80%.

Ex 2: (a) Translate decimal 0.079 as percentage

(b) Translate percentage of 11.2% as a decimal

(a) $0.079 = 7.9\%$

(b) $11.2\% = 0.112$ ▫

If the value of some goodies increased by x%, it means that

$$\boxed{(\text{new value}) = (\text{old value}) \times \left(1 + \frac{x}{100}\right)}$$

In particular, if the value increased once by x% and then next month the value increased again by y%, then to compute the new value, we multiply the initial value by $\left(1+\frac{x}{100}\right)\cdot\left(1+\frac{y}{100}\right)$

! Warning: In particular, $\left(1+\frac{x}{100}\right)\cdot\left(1+\frac{y}{100}\right) = 1 + \frac{x+y}{100} + \frac{xy}{10000} > 1 + \frac{x+y}{100}$

Hence the total value increased by more than $(x+y)\%$ from its original value

We will return to that issue when discussing compound interests.