Lecture #25

• Last time: Compound Interest

Key Formula: $\boxed{F = P\cdot(1+r)^t}$

Rem(1) If the interest is compounded semi-annualy / quarterly / monthly / daily, while your interest is given as an APR, then for r you should take $\frac{APR}{2}$ / $\frac{APR}{4}$ / $\frac{APR}{12}$ / $\frac{APR}{365}$, respectively.

(2) "Rule of 72": It takes approximately $\frac{72}{APR}$ years to double investment.

• Continuous Compounding

Last time, we mentioned a limit case of compound interest, a.k.a. "continuous compounding". In other words, the interest is compounded infinitely often over infinitely small compounding periods.

While, we leave the mathematically rigorous proof for your future Calculus class (if you take it), let me write down the final formula:

$$\boxed{F = P\cdot e^{rt}}$$

Here $e \approx 2.718281828\ldots$ - the Euler's Number (popular in calculus :)

You definitely will need calculator for computing F explicitly.

Let us consider the same set-up as we had on Wedn in the last Example.

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Ex1: John invested \$2,000 into a CD with an APR of 3.6% compounded continuously. Find the future values after 1, 5, 10, 15, 20 years.

▸ $F = \$2000 \cdot e^{0.036t}$. Use CALCULATOR!

$t=1 \Rightarrow F \approx \$2073^{31}$

$t=5 \Rightarrow F \approx \$2394^{43}$

$t=10 \Rightarrow F \approx \$2866^{66}$

$t=15 \Rightarrow F \approx \$3432^{01}$

$t=20 \Rightarrow F \approx \$4108^{87}$ ▪

Rmk: Comparing last numeric data (corresp. to $t=20$) with those from Wedn, we see that after 20 years John will get:

- $\approx 4057^{13}$ \$ when interest is compounded annualy
- $\approx 4104^{44}$ \$ when interest is compounded monthly
- $\approx 4108^{87}$ \$ when interest is compounded continuously.

In particular, the difference b/w the last two amounts is very small.

• Annual Percentege Yield (APY)

APY is the annual percentage increase on the value of an investment (or loan) when both APR and frequency of compounding are taken into account.

* If APR is r and interest is compounded N times per year $\Rightarrow$ $\boxed{APY = \left(1+\frac{r}{N}\right)^N - 1}$
* If APR is r and interest is compounded continuosly $\Rightarrow$ $\boxed{APY = e^r - 1}$

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Ex2 (Exercise 10.3.47): Find the APY for an APR of 3% compounded (a) yearly, (b) semi-annualy, (c) monthly, (d) continuously

(a) Yearly $\Rightarrow$ APY = APR = 3%

(b) $APY = \left(1+\frac{3}{100}\cdot\frac{1}{2}\right)^2 - 1 = 3.0225\%$

(c) $APY = \left(1+\frac{3}{100}\cdot\frac{1}{12}\right)^{12} - 1 \approx 3.0416\%$

(d) $APY = e^{0.03} - 1 \approx 3.0455\%$

• "Consumer Debt" - Section 10.4

Ex3 (Exercise 10.4.51) Elizabeth left a balance of \$1,200 on her credit card after all her expenses in June. Hence, she started a new billing cycle June 19 - July 18 with a previous balance of \$1,200 (and with interest APR of 19.5%). In addition she made three purchases with the dates and amounts shown below, while on July 15 she made an online payment of \$500.00

Date	Purchase/Payment
6/21	\$179.58
6/30	\$40.00
7/5	\$98.35
7/15	Payment of \$500.00

(a) Find the average daily balance for this billing cycle

(b) Compute the interest charged for this cycle.

(c) Find the new balance on the account at the end

(a) June 19-20: Balance of \$1200

June 21-29: Balance of \$1200 + \$179.58 = \$1379.58

June 30-July 4: Balance of \$1379.58 + \$40.00 = \$1419.58

July 5-14: Balance of \$1419.58 + \$98.35 = \$1517.93

July 15-18: Balance of \$1517.93 - \$500.00 = \$1017.93

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(Continuation)

$$\text{Average Daily Balance} = \frac{2\cdot 1200 + 9\cdot 1379.58 + 5\cdot 1419.58 + 10\cdot 1517.93 + 4\cdot 1017.93}{30} \$$$

$$= \frac{2400 + 12416.22 + 7097.9 + 15179.3 + 4{,}071.72}{30} \$ \approx 1372^{17} \$$$

(b) Interest rate for this billing cycle : $\frac{30}{365}\cdot 0.195$

Hence, interest charged is

$$\frac{30}{365}\cdot 0.195 \cdot 1372^{17} \$ \approx 21^{99} \$$$

(c) New balance will be $1017^{93} + 21^{99} = 1039^{92} \$$

- Installment loans ← The Borrower pays the loan plus interest back to the lender in equal installments paid at regular intervals.

This process is called amortization, while the loan is said to be amortized over a certain period of time.

In the US, almost all installment loans are based on monthly payments.

Amortization Formula : $\boxed{M = P\cdot \frac{p(1+p)^N}{(1+p)^N - 1}}$

here r = APR, N = total number of months over which installments are paid, P = Principal, M = monthly payment, $p = \frac{r}{12}$

Ex 4 (Exercise 10.4.53) You purchase a car and finance \$14,500 for 36 months at APR of 6% compounded monthly. Find monthly payments.

$P = 14{,}500\$$, $r = 0.06 \Rightarrow p = \frac{r}{12} = 0.005$, $N = 36 \Rightarrow M = 14500\cdot\frac{0.005\cdot 1.005^{36}}{1.005^{36}-1} \approx 441^{12}$

! If time permits, give a proof of the above amortization formula.