Lecture #26 10/31/2016

- Discuss Ex3 from pp. 3-4 of Lecture Notes #25
  Provide detailed arguments

- Discuss Amortization Formula and 1 example on applying it from p. 4 of Lecture Notes #25.

Amortization Formula: $\boxed{M = P \cdot \frac{p \cdot (1+p)^N}{(1+p)^N - 1}}$

Q: Why do we have such a strange formula?
Let me actually derive it from what we already learnt
(this was not done in the class and will not be needed for exams).

Step 0: You take a loan of \$P, i.e. your original balance is $B_0 = \$P$

Step 1: After 1st month you owe $\$P + \underbrace{p \cdot \$P}_{\text{interest due}} = \$P \cdot (1+p)$
But you also make a payment of \$M after 1st month
Hence, your balance due after 1st month is $\$P \cdot (1+p) - \$M =: B_1$

Step 2: After 2nd month you owe $B_1 + \underbrace{p \cdot B_1}_{\text{interest due after 2nd month}}$
But you also make a payment of \$M at the end
$\Rightarrow$ your Balance due after 2nd month is $B_2 = \$B_1(1+p) - \$M$

$\vdots$

Step N: After N^{th} month your balance due is $B_N = B_{N-1}(1+p) - \$M$

So: $B_N = B_{N-1}(1+p) - M = (B_{N-2}(1+p) - M)(1+p) - M = ((B_{N-3}(1+p) - M)(1+p) - M)(1+p) - M$ etc...

$= \ldots = B_0 \cdot (1+p)^N - M - M(1+p) - M(1+p)^2 - \ldots - M(1+p)^{N-1}$

Geometric Sum F-la: $1 + (1+p) + (1+p)^2 + \_\_ + (1+p)^{N-1} = \frac{(1+p)^N - 1}{(1+p) - 1} = \frac{(1+p)^N - 1}{p}$

(Continuation)

So, we found that

$$B_N = B_0 \cdot (1+p)^N - M \cdot \left(1 + (1+p) + (1+p)^2 + \_\_\_ + (1+p)^{N-1}\right)$$

$$= P \cdot (1+p)^N - M \cdot \frac{(1+p)^N - 1}{p}$$

But on the other hand, you have to pay back all the loan + interest after N months, i.e. $B_N = 0$.

Thus: $0 = P \cdot (1+p)^N - M \cdot \frac{(1+p)^N - 1}{p}$

$$\Downarrow$$

$$\boxed{M = P \cdot \frac{p(1+p)^N}{(1+p)^N - 1}}$$

Rmk: In real life, if you google "Amortization Formula", you will find many online calculators which compute M given P, p, N. But now you should know where it is coming from.