

- Discuss Problems 1 & 3 from Midterm 2
- Hand out Questionnaires about Midterm
- New topic "Rigid Motions" (Section 11 of your Textbook)

Def: A rigid motion is an act of taking an object and moving it from the starting position to some ending position without altering its shape/size.

Note: The distance between any 2 points is preserved under applying a rigid motion.

Def: Given an initial object (starting position of it), we say that two rigid motions are equivalent if they "move" the object to the same ending position.

Notations: • We will use curly letters, e.g. M, N, A, B, \dots to denote rigid motions

• Given a point P of an object and a rigid motion M , we will denote the ending position of P by $\underline{P'}$ and call it the image of P under the rigid motion M .

Def: Given a rigid motion M and a point P , we say that P is a fixed point of the rigid motion M if $P' = P$.

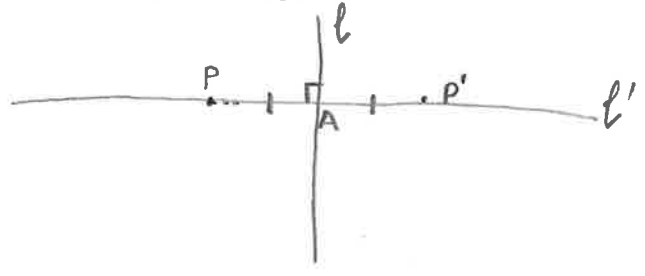
• For 2-dimensional objects moving in a plane, there are only 4 types of rigid motions and we will discuss this next.

1st type: "Reflections"

A reflection in the plane is a rigid motion that moves an object into a new position that is a mirror image of the starting position.

The role of a mirror is played by a line called the axis of reflection.

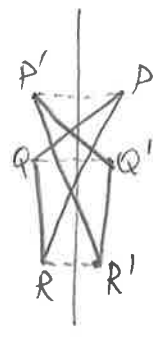
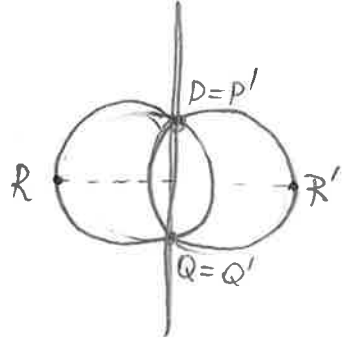
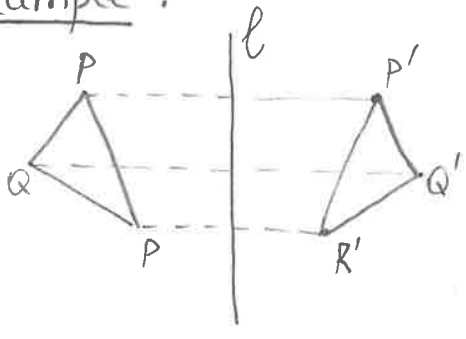
In other words, given any line l (= the axis of reflection), we consider reflection of any point. To do that, consider a line l' perpendicular to l and passing through P .



Then P' is also on that line, on the opposite side of l , so that the distances $PA = P'A$

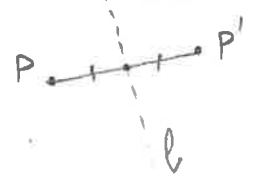
(A = intersection point of l & l')

Example:



Key Observation

You can recover an axis of reflection l given any point P and its image P' such that $P' \neq P$. For this you just need to construct a line passing through the middle-point of PP' and perpendicular to PP'



Key Properties of Reflections

1. Reflection is completely determined by its axis l .
2. Reflection is completely determined by a point P and its image P' (assuming $P' \neq P$)
3. Fixed points of a reflection are exactly the points of its axis.
4. Applying the same reflection twice, we always get the same position of an object as we started from.

In other words, $\text{Reflection}^2 = \text{Identity map}$

5. Reflections are improper rigid motions
changes orientation.

• Discuss several examples in class.