- Briefly remind 3 different types of rigid motions of a plane we learned last time:
  - reflection
  - rotation
  - translation

Give a few examples illustrating these 3 motions.

<u>Thm1</u>: Any rigid motion of a plane is a composition of the above simplest motions.

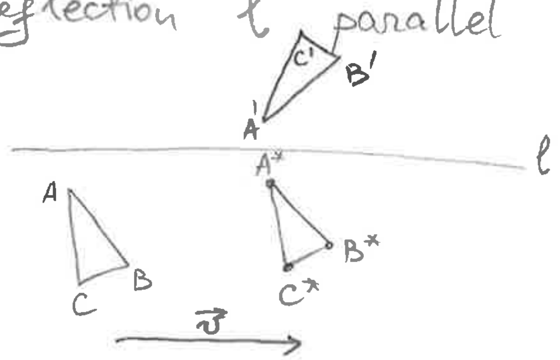
! Explain what a "composition" means.

<u>However</u>, the result of this theorem can be rephrased in a better form:

<u>Thm2</u>: Any rigid motion of a plane is one of the following 4 options: (1) reflection, (2) rotation, (3) translation, (4) glide reflection.

A glide reflection is a rigid motion obtained by combining a translation (by vector $\vec{v}$) with a reflection with an axis of reflection l parallel to $\vec{v}$

<u>Ex</u>:



This result (Thm2) concludes our discussion of rigid motions.

Lecture #29 11/09/2016

- Today: Starting Chapter 13 "Fibonacci numbers and Golden ratio"

We start our discussion with a Fibonacci sequence.

Problem (Fibonacci's rabbits): A certain man put a pair of rabbits in a place surrounded in all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on becomes productive?

- Start (time $=0$): $P_0=1$ (1 pair given initially).
- Month 1 (time $=1$): $P_1=1$ but now they can produce offspring
- Month 2 (time $=2$): $P_2=2$ (original pair + baby pair)
- Month 3 (time $=3$): $P_3=3$ and 2 pairs of those 3 are mature
- Month 4 (time $=4$): $P_4=5$ and 3 pairs of those 5 are mature to produce offspring

  ⋮
- Month N : $P_N = P_{N-1}+P_{N-2}$

So: we recover the Fibonacci sequence we already discussed some time ago:

$$\boxed{P_0=1,\ P_1=1,\ P_N=P_{N-1}+P_{N-2} \text{ for any } N\geq 2.}$$

Step-by-step:

$P_0=1$, $P_1=1$, $P_2=2$, $P_3=3$, $P_4=5$, $P_5=8$, $P_6=13$, $P_7=21$, $P_8=34$, $P_9=55$, $P_{10}=89$, $P_{11}=144$, $\boxed{P_{12}=233}$

Answer: There are 233 pairs of rabbits after 1 year ∎

Rmk: Here we used the recursive f-la defining the Fibonacci sequence

Q: Is there an explicit formula for the N-th Fibonacci number?

Lecture #29

11/09/2016

- Binet's formula

Let $F_1, F_2, \ldots$ be the Fibonacci sequence, i.e. $F_1 = F_2 = 1$, $F_N = F_{N-1} + F_{N-2}$.
The explicit f-la for F_N was first discovered by Leonhard Euler in 1736 and was rediscoverd a century after that by Jacques Binet

$$\boxed{\text{Binet's Formula: } F_N = \left[\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N\right]/\sqrt{5}}$$

Taking into account the fact that $-1 < \frac{1-\sqrt{5}}{2} < 0$, the above f-la can be simplified to the formula we already saw when discussing Fibonacci numbers first:

$$\boxed{\text{Simplified Binet's Formula: } F_N = \left[\!\left[\left(\frac{1+\sqrt{5}}{2}\right)^N / \sqrt{5} \right]\!\right]}$$

N.B.: $[\![\cdot]\!]$ denotes the nearest integer

Rmk: For those of you who is curious why those crazy numbers $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$ come into the play, let me just mention that those are the roots of the "associated characteristic" polynomial $x^2 - x - 1 = 0$.

* Surprisingly, Fibonacci numbers show up in natural organisms. See pictures on p. 393 of your textbook illustrating examples of petals of daisies, bracts of a pinecone, seeds on a sunflower head.