Lecture #31 11/14/2016

- Start from Ex3 from Lecture #30: find solutions of $F_N \cdot x^2 = F_{N-1} \cdot x + F_{N-2}$ (Discuss in details)
- Today: "Gnomons"

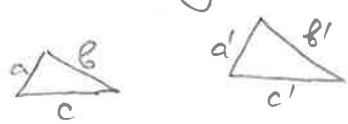
Two objects are said to be similar if one is a scaled version of the other.

Examples:

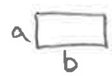
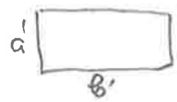
(1) Any two squares are similar 

(2) Any two disks are similar 

(3) Two triangles with sides a, b, c $(a \le b \le c)$ and a', b', c' $(a' \le b' \le c')$ are similar if and only if there exists a scaling factor $k > 0$ such that $\boxed{a' = ka,\ b' = kb,\ c' = kc}$



Rmk: They are also similar if and only if they have the same angles.

(4) Two rectangles  ,  with sides a, b $(a \le b)$ and a', b' $(a' \le b')$ are similar if and only if there exists a scaling factor $k > 0$ such that $\boxed{a' = ka,\ b' = kb}$ or equivalently $\boxed{\frac{a}{b} = \frac{a'}{b'}}$

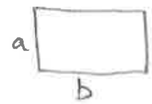
Rmk: We assume $a \le b$, $a' \le b'$ or otherwise we could also have $a' = kb$, $b' = ka$.

Key definition: A gnomon G to a figure A is a connected figure which when suitably attached to A produces a new figure similar to A. We use A&G to denote a figure obtained by attaching G to A.

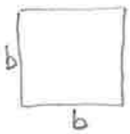
Example 1: If A =  - the disk of radius r, then G =  is obviously a gnomon to A, since A&G =  - the disk of radius R

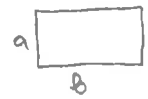
Lecture #31 11/14/2016

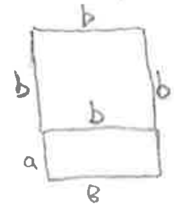
Q: Which rectangles have square gnomons?

Consider a rectangle $R =$ 

and assume that the square

$G =$ 

is a gnomon of R, i.e. 

is similar to 

Then $\frac{b}{a} = \frac{b+a}{b}$ $\Rightarrow$ the sides of R are in a divine proportion

(see Lecture Notes #30)

In other words, $\frac{b}{a} = \phi$ - the golden ratio. $\rightsquigarrow$ $\boxed{R \text{ is a golden rectangle}}$

! **Rmk**: Given two figures A and B such that B is a scaled version of A with the scaling factor $k > 0$, we have:

(a) Perimeter $(B) = k \cdot$ Perimeter (A)

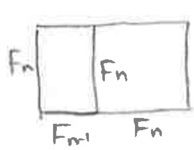
(b) Area $(B) = k^2 \cdot$ Area (A)

Discuss these 2 properties!!!

Above we defined the Golden rectangles.

Analogously, one can define Fibonacci rectangles as rectangles whose sides are consecutive Fibonacci numbers.

Note: Starting from a Fibonacci rectangle with sides F_{n-1} and F_n, and attaching to it a square of size $F_n \times F_n$, we get a Fibonacci rectangle with sides F_n and F_{n+1}



$(F_{n-1} + F_n = F_{n+1})$