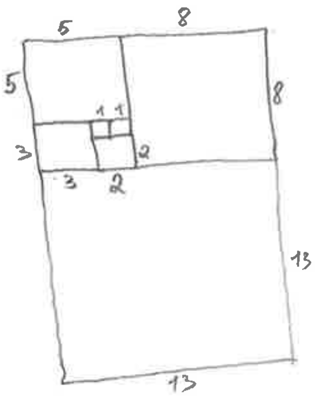
Lecture #32 11/16/2016

• Last time: Golden and Fibonacci rectangles

* Recall their definitions

* Consecutive Fibonacci rectangles



We start from the smallest Fibonacci rectangle $1\times 1 = F_1 \times F_2$ and attach to it squares step by step. Thus we will obtain all the Fibonacci rectangles.

Fact: $F_1^2 + F_2^2 + F_3^2 + \_\_ + F_N^2 = F_N \cdot F_{N+1}$

This immediately follows from the above picture, just by computing the area of the Fibonacci rectangle in two different ways.

E.g. consider $N=7$. Then the rectangle above has size $13\times 21 = F_7 \times F_8$ and hence its area is equal to $F_7 \cdot F_8$. On the other hand, it consists of squares 1×1, 1×1, 2×2, 3×3, 5×5, 8×8, 13×13 and, therefore, its area is equal to $F_1^2 + F_2^2 + F_3^2 + F_4^2 + F_5^2 + F_6^2 + F_7^2$.

So: $F_1^2 + F_2^2 + \_\_ + F_7^2 = F_7 \cdot F_8$.

Same argument works for arbitrary N ■

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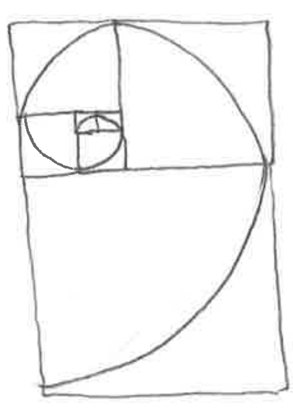
* Spiral growth

Let us start by observing that the ratio of the sides of the Fibonacci rectangle $F_N \times F_{N+1}$, defined as $\frac{F_{N+1}}{F_N}$, approaches to the golden ratio ϕ as already discussed.
In particular, they become very close to the golden rectangles, hence, they are almost similar between themselves.

We may think of this "attaching of square" procedure as a certain growth model of a living organism, i.e. on each generation we add a square. Then the above observation implies that the overall shape of an organism stays almost the same.

! There are some examples of organisms, where the growth is obtained just by adding a new part to an old organism.
Examples are: shell of a chambered nautilus, ram's horn, (see p. 402 of Textbook) trunk of a redwood tree.

Aside: Fibonacci spiral is a beautiful spiral obtained by drawing quarter-circle in each of the squares from p. 1:



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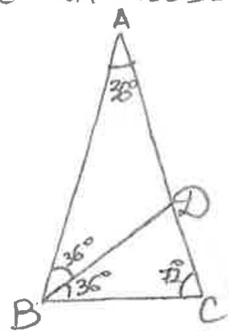
* Golden Triangles

A similar discussion and picture can be made in the setting of isosceles triangles.

Observation: Consider an isosceles triangle

and bisect the left-bottom angle.

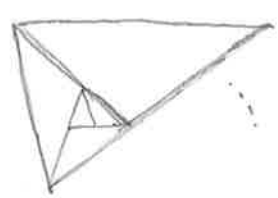


Then the triangle BCD has the same angles 36°, 72°, 72° as ABC. Hence they are similar.

On the other hand, ABD is also an isosceles triangle with angles 36°, 36°, 108°.

So: ABD is a gnomon to BCD.

This leads to the following picture (spiraling series of ever increasing similar triangles).

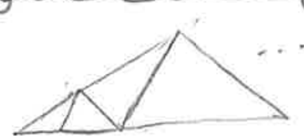


Rmk: $AC = AB$, $BD = AD = BC \Rightarrow \dfrac{AC}{AD} = \dfrac{AD}{AC} \Rightarrow$ divine propoption equal to ϕ.

So: $\dfrac{AC}{BC} = \dfrac{AC}{AD} = \phi$, i.e. the ratio of sides of ABC is equal to the golden ratio, hence, the name "golden triangle"

Likewise, $\dfrac{AB}{AD} = \phi \Rightarrow$ the triangle with angles 36°, 36°, 108° is also a golden triangle.

(Moreover, we also get a chain-series of ever increasing similar triangles with angles 36°, 36°, 108°.)



Lecture #32

11/16/2016

- New Topic "Probabilities, Odds, and Expectations" (Sect. 16)

• Random experiment – activity/process whose outcome cannot be predicted.

Examples: Tossing a coin, rolling a pair of dice.

Def: Sample space (of the experiment) is the set of all possible outcomes.

! For simplicity, we will consider only examples with finite sample sets, though in real life there are many examples with a sample space being infinite.

Notation: Sample space will be denoted by S, while N will be the size of S, i.e. $N = \#$ elements in S.

Ex 1.1 (Tossing a coin once): Tossing a coin once, we have two options: head or tail. Hence, $S = \{T, H\}$ and $N = 2$

Ex 1.2 (Tossing a coin twice): In this case, the sample space is $S = \{TT, TH, HT, HH\}$ and $N = 4$.

Ex 1.3 (Same as Ex 1.2, but we only care about the total number of heads): There are 3 possible outcomes: 0 heads, 1 head, 2 heads in total. Hence, $S = \{0, 1, 2\}$ and $N = 3$.