- Last time: Started discussion of Section 16 on probability
  - S - the sample space
  - N - size of S, i.e. number of elements in S.
  - Discussed 3 examples about tossing a coin
- Let us consider another example involving a pair of dice

Ex 1.1 (Rolling a pair of dice): There are 6 possibilities (1, ..., 6 dots) for each of the two dice. Hence the sample set S consists of all pairs $\{(a,b) \mid a=1,\dots,6;\ b=1,\dots,6\}$. Hence $N = 6 \cdot 6 = 36$

Ex 1.2 (Same experiment, but we only care about the total sum of 2 numbers)
Since $1 \le a \le 6$, $1 \le b \le 6$, we see that $1+1=2 \le a+b \le 12 = 6+6$ and $a+b$ - integer
Hence, $S = \{2, 3, \dots, 11, 12\}$ and $N = 11$

[Rem (relevant to hwk): The last example from last time can be interpreted as follows. A coin is tossed twice. The observation is the percentage of tosses that are Heads. Write out the sample space.
Clearly $S = \{0\%, 50\%, 100\%\}$, where $0\% \Leftrightarrow$ No Heads, $50\% \Leftrightarrow$ 1 Head, 100% - All Heads]

- Another standard set of examples concerns drawing cards out of a deck

Ex 2: A card is drawn out of the standard deck of 52 cards. Each card is described by giving its value (A, 2, 3, $\rightarrow$ 10, J, Q, K) and its suit (H = hearts, C = clubs, D = diamonds, S = spades)
Write out the event described by each of the following statements as a set:

(a) E_1: "draw a diamond"

(b) E_2: "draw a king"

• Event is a subset of the sample space

Recall: The finite set on N elements has 2^N different subsets

So: There are 2^N different events.

- empty set $\{\}$ represents an event with no outcomes, which we call impossible event
- entire space S (viewed as a subset of S) represents an event, where all possible outcomes are included, called certain event
- 1-element subsets represent event consisting of a single outcome, called simple event

Ex3: Discuss the above concepts in the experiment of tossing a coin twice.

• Combinatorics

Goal/Motivation: Compute the number of outcomes without having to list them all.

(Rule #1) Multiplication Rule: If there are m different ways to do X and n different ways to do Y, then X & Y together can be done in $m \cdot n$ diff ways.

Ex 4: (a) Tossing a coin 5 times, find the total number of outcomes

(b) Rolling 3 dice, find the total number of outcomes

(c) Point out the #cards in a deck is equal to $13 \times 4 = 52$.

(a) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$

(b) $6 \cdot 6 \cdot 6 = 6^3 = 216$

(c) tautological

Lecture #33 11/18/2016

Ex 5 (Exercise 16.2 19): Jack packs two pairs of shoes, one pair of boots, three pairs of jeans, four pairs of dress pants, and three dress shirts for a trip.

(a) How many different outfits can Jack make with these items?

(b) If Jack were to bring along two sweaters so that he could wear either a dress shirt or a sweater, or both a dress shirt and a sweater, how many outfits could Jack make?

► We use the box model for each of these countings.

(a)

$$\boxed{\overbrace{\underset{\text{shoes}}{2}+\underset{\text{boots}}{1}}^{3}} \times \boxed{\overbrace{\underset{\text{jeans}}{3}+\underset{\text{dress pants}}{4}}^{7}} \times \boxed{\underset{\text{dress shirts}}{3}} = 3\cdot 7\cdot 3 = \underline{\underline{63}}$$

(b)

$$\boxed{\overbrace{2+1}^{3}} \times \boxed{\overbrace{3+4}^{7}} \times \boxed{\overbrace{\underset{\text{shirt}}{3}+\underset{\text{sweater}}{2}+\underset{\text{shirt \& sweater}}{3\cdot 2}}^{11}} = 3\cdot 7\cdot 11 = \underline{\underline{231}}$$

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* Permutations and Combinations

Goal: Count groups of objects, either ordered or not.

Ex 6: How many outcomes in rolling a pair of dice have two different numbers as an outcome?

► 1^{st} dice : 6 options. For each of them, 2^{nd} dice: $6-1=5$ options

$$\boxed{6} \times \boxed{5} = \underline{\underline{30}}$$

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! If we were only caring about the set of values on those dice, we would get $\frac{1}{2}\cdot 30 = 15$ as the order is irrelevant now.

Ex 7: Upgrade to three dice. Same question.

$6\cdot 5\cdot 4 = 120$ if ordered

$\frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1} = 20$ if unordered

} give more details in class

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General Setting

We start from a set of $\underline{n}$ elements and want to choose $\underline{r}$ different elements from this set. The choices can be either ordered or unordered.

Permutation: An ordered selection of r objects from a set of n objects. Here $0 \le r \le n$.

Combination: An unordered selection of r objects chosen from a set of $\underline{n}$ objects (again $0 \le r \le n$).

Notation: (a) ${}_nP_r$ = number of permutations of r objects chosen from a set of n objects

(b) ${}_nC_r$ = number of combinations of r objects chosen from a set of n obj-s.

Key Formulas:

$$\boxed{\begin{aligned} {}_nP_r &= n\cdot(n-1)(n-2)\cdot \ldots \cdot(n-r+1) = \frac{n!}{(n-r)!} \\ {}_nC_r &= \frac{{}_nP_r}{r!} = \frac{n!}{r!\,(n-r)!} \end{aligned}}$$