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Ex 1: (a) How many outcomes in rolling four dice have 4 different numbers as an outcome?

(b) Same question, but now we are interested only in possible collections of those 4 different numbers

Ex 2: Discuss example Ex2 from Lecture Notes #33

General Setting

We start from a set of $\underline{n}$ elements and want to choose $\underline{r}$ different elements from this set. The choices can be either ordered or unordered.

Permutation: An ordered selection of r objects from a set of n objects. Here $0 \le r \le n$.

Combination: An unordered selection of $\underline{r}$ objects chosen from a set of $\underline{n}$ objects (again $0 \le r \le n$).

Notation: (a) ${}_nP_r$ = number of permutations of r objects chosen from a set of n objects

(b) ${}_nC_r$ = number of combinations of r objects chosen from a set of n obj-s.

Key Formulas:

$$\boxed{\begin{aligned} {}_nP_r &= n\cdot(n-1)(n-2)\cdot \ldots \cdot(n-r+1) = \frac{n!}{(n-r)!} \\ {}_nC_r &= \frac{{}_nP_r}{r!} = \frac{n!}{r!\,(n-r)!} \end{aligned}}$$

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Ex3 (Exercise 16.2.23 / 24) How many seven-digit numbers

(a) are even?

(b) are divisible by 5?

(c) are divisible by 25?

(d) have no repeated digids?

(e) are number palindromes?

↑ same number as if we read left-to-right or right-to-left

Ex 4 (Exercise 16.2.22) Nine people (4 men and 5 women) line up at a check-out stand in a grocery store

(a) In how many ways can they line up?

(b) In how many ways can they line up if the first person in line must be a woman?

(c) In how many ways can they line up if they must alternate woman, man, woman, man, ... ?

- Probability

Ex 5 (Baby-example):

(a) Tossing a coin once, what is the probability to get Head?

(b) Tossing a coin 10 times, what is the probability to get all Heads?

(c)* Tossing a coin 3 times, what is the probability to have more Heads than Tails?

(Answer: (a) $1/2$, (b) $1/2^{10} = 2^{-10}$, (c) $1/2$)

However, in real life, it is not always the case that each outcome has the same probability.

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A **probability assignment** is a function that assigns to each event E a number between 0 and 1, which represents the probability of the event E, denoted $Pr(E)$. We always assume $Pr(\{\})=0$, $Pr(S)=1$.

! If S is finite, then it suffices to assign probability only to simple events, i.e. 1-element subsets of the sample space. Indeed, we recover $Pr(E)$ just by summing up the probabilities of the individual outcomes that make up the event E.

Upshot: To define Pr, it suffices to assign a number from 0 to 1 to each simple event, so that these numbers add up to 1.

Probability space is a combination of a sample space and the probability assignment.

- Sample space $S=\{O_1,\dots,O_N\}$
- Probability assignment: $0\le Pr(O_1),\dots,Pr(O_N)\le 1$ such that $Pr(O_1)+\dots+Pr(O_N)=1$
- Event $E=\{O_{i_1},\dots,O_{i_k}\}\subset S$ has probability $Pr(E):=Pr(O_{i_1})+\dots+Pr(O_{i_k})$.

Ex 5 (Exercise 16.3.33) Consider the sample space $S=\{O_1,O_2,O_3,O_4,O_5\}$. Suppose $Pr(O_1)=0.22$, $Pr(O_2)=0.24$.

(a) Find the probability assignment for the probability space when O_3,O_4,O_5 all have the same probability

(b) Find the probability assignment for the probability space when $Pr(O_5)=0.1$, and $\{O_3\}$ has the same probability as $\{O_4,O_5\}$.