Lecture #35

- Last time:
  - $_nP_r$ = number of permutations on r objects from a set of n objects

    $=$

    $n \cdot (n-1) \cdots (n-r+1) = \dfrac{n!}{(n-r)!}$

  - $_nC_r$ = number of combinations of r objects chosen from a set of n objects

    $=$

    $\dfrac{_nP_r}{r!} = \dfrac{n!}{(n-r)! \, r!}$

Ex1 (Exercise 16.2.22): Nine people (4 men and 5 women) line up at a check-out stand in a grocery store

(a) In how many ways can they line up?

(b) In how many ways can they line up if the first person in line must be a woman?

(c) In how many ways can they line up if they must alternate (woman, man, woman, man,...)?

➤ (a) $9!$ (b) $5 \cdot 8!$ (c) $5! \cdot 4!$

Ex2 (Exercise 16.2.32): Bob has 20 different dress shirts in his wardrobe.

(a) In how many ways can Bob select seven shirts to pack for his business trip?

(b) In how many ways can Bob select 5 of those 7 dress shirts he packed: one for Monday, one for Tue,..., one for Friday?

➤ (a) $_{20}C_7 = \dfrac{20!}{7! \cdot 13!} = 77,520$ (b) $_7P_5 = \dfrac{7!}{2!} = 2520$ ∎

- Last time : Probability assignment

  * Ask the audience if anyone remembers the definition.

  Recall : Probability assignment is a function that assigns to
      each event E a number between 0 and 1 , which
      represents the probability of the event E, denoted $Pr(E)$

Properties : (1) $Pr(\{\}) = 0$

           (2) $Pr(S) = 1$

           (3) $Pr(E \cup E') = Pr(E) + Pr(E')$ if $E \cap E'$ is empty.

! Due to the property (3), if S is finite , i.e. $S = \{x_1, \dots, x_N\}$,
then Pr is uniquely determined by specifying

     $0 \le Pr(\{x_1\}), \dots, Pr(\{x_N\}) \le 1$ such that $Pr(\{x_1\}) + \dots + Pr(\{x_N\}) = 1$

Then the value $Pr(E)$ is obtained summing up the probabilities
of the individual outcomes that make up the event.

Def : The combination of the sample space S and the
     probability assignment is called a probability space.

Ex3 (Exercise 16.3.33): Consider the sample space
  $S = \{x_1, x_2, x_3, x_4, x_5\}$. Suppose $Pr(\{x_1\}) = 0.22$, $Pr(x_2) = 0.24$.

(a) Find the probability assignment for the probability
    space when x_4, x_5, x_6 all have the same probability.

(b) Find the probability assignment for the probability
    space when $Pr(x_5) = 0.1$ and $\{x_3\}$ has same probability as $\{x_4, x_5\}$

Lecture #35

- New material for today

‖ Def: A probability space in which each simple event has an
    equal probability is called an equiprobable space.
If N is the size of the sample space, then we see that each
individual outcome has probability $1/N$. More generally,
if $k =$ size of E, $N =$ size of S, then in an equiprobable space

$$\boxed{P_r(E) = k/N.}$$

Ex4 (Exercise 16.3.39) An honest coin is tossed three times in
                        a row. Find the probability of the
                        following events.

(a) E_1: "the coin comes up heads exactly twice"

(b) E_2: "all three tosses come up the same"

(c) E_3: "exactly half of the tosses come up heads"

(d) E_4: "the first two tosses come up tails".

▶ (a) $3/8$,   (b) $2/8 = 1/4$,   (c) 0,   (d) $2/8 = 1/4$.

! It is also very instructive to consider Example 16.21
from your textbook.