- Last time: "indepent events" and "odds of / against the event".

Ex1: An honest coin is tossed 10 times. Find the probability of:

(a) E_1: "a tail comes up at least once"

(b) E_2: "a tail comes up at least twice".

(a) $1 - 1/2^{10} = 1023/1024$

(b) $1 - \frac{1+10}{2^{10}} = 1013/1024$ ∎

Ex2: (a) Find the odds of the event E with $Pr(E)=0.1$

(b) Find the probability of an event E with odds in favor of E are 9:11.

(a) 1:9

(b) $\frac{9}{20} = 0.45$ ∎

- Today: "Expectations"

Def: Let X be a variable that takes the values $x_1, \dots, x_N$ and let $w_1, \dots, w_N$ denote the respective "weights" for these values, with $w_1 + \dots + w_N = 1$. The weighted average for X is given by

$$w_1 \cdot x_1 + w_2 \cdot x_2 + \_\_\_ + w_N \cdot x_N.$$

Remk: In most examples we will be interested in, the weights represent probability.

Lecture #37

12/05/2016

Ex3 (Exercise 16.4.59) At T.J. high school, the student body is divided by age as follows:

7% of students are 14,
22% of students are 15,
24% of students are 16,
23% of students are 17,
19% of students are 18,
rest of students are 19.

Find the average age of the students at TJ high school.

- Since the sum of % should be 100%, we see that $(100-7-22-24-23-19)\% = 5\%$ of students are 19.
- Average age $= 0.07\cdot 14 + 0.22\cdot 15 + 0.24\cdot 16 + 0.23\cdot 17 + 0.19\cdot 18 + 0.05\cdot 19$
$= \boxed{16.4}$

Ex 4 (Example 16.30): On a test, all q-s are multiple-choice with five possible answers in each problem. You get +1pt or −1/4 pt for correct/wrong

(a) Assuming you can't eliminate any of the answers and decide on random guessing. Would it hurt you or benefit you?

(b) Assuming you can rule out one of the five choices. Would it hurt you or benefit you if you decide to guess randomly the answer out of the remaining 4 options. "Measure" your answer.

(a) Expected payoff is $1\cdot 0.2 - \frac{1}{4}\cdot 0.8 = 0$ ⇒ wouldn't hurt nor benefit

(b) Expected payoff is $1\cdot 0.25 - \frac{1}{4}\cdot 0.75 = 0.0625$ ⇒ would benefit you

The outcome from Ex 4, is that you should guess the answer if you can rule out several options.

Def: Suppose X is a variable that takes numerical values $x_1, \ldots, x_N$ with probabilities $p_1, p_2, \ldots, p_N$ respectively. The expectation (aka expected value) of X is given by

$$\boxed{E = p_1 \cdot x_1 + p_2 \cdot x_2 + \ldots + p_N \cdot x_N}$$

! If X is equiprobable, i.e. $p_1 = p_2 = \ldots = p_N = \frac{1}{N}$, then we get the "standard" average of $x_1, x_2, \ldots, x_N$.

Next time we will see how this notion appears in real-life situations which require decision making. In other words, computing the expectation we may decide if certain reward is worth the risk.

But before we switch to this, let us consider one example related to your own experience.

Ex 5: The grading for MAT 118 is based on Hwk, 2 Midterms, Final. The breakdown for Bob's scoring is given in the table:

	Hwk	Midt1	Midt2	Final
Weight	25%	20%	20%	35%
Possible points	200	100	100	200
Bob's score	160	87	92	170

Compute Bob's average.

$$\frac{160}{200} \cdot 25\% + \frac{87}{100} \cdot 20\% + \frac{92}{100} \cdot 20\% + \frac{170}{200} \cdot 35\% = 85.55\%$$

So Bob's average in that class is 85.55%. ∎