Lecture # 38

12/07/2016

- Last time: weighted average

Ex 1: See Ex 4 from Lecture # 37.

- The notion of a weighted average leads to the following important concept in Probability:

Def: Suppose X is a variable that takes numerical values $x_1, \ldots, x_N$ with probabilities $p_1, \ldots, p_N$, respectively. The expectation (a.k.a. expected value) of X is given by

$$E = p_1 \cdot x_1 + \ldots + p_N \cdot x_N$$

Rmk: If X is equiprobable, i.e., $p_1 = p_2 = \ldots = p_N = \frac{1}{N}$, then we recover the standard notion of an average you should be familiar with.

Ex 2 (Exercise 16.4.61): A box contains twenty \$1 bills, ten \$5 bills, five \$10 bills, four \$20 bills, and one \$100 bill. You blindly reach into the box and draw a bill at random. What is the expected value of your draw?

$$\left.\begin{array}{l} E = p_1 \cdot \$1 + p_2 \cdot \$5 + p_3 \cdot \$10 + p_4 \cdot \$20 + p_5 \cdot \$100 \\ \text{Total number of bills} = 20+10+5+4+1 = 40 \\ \Rightarrow p_1 = \frac{20}{40},\ p_2 = \frac{10}{40},\ p_3 = \frac{5}{40},\ p_4 = \frac{4}{40},\ p_5 = \frac{1}{40} \end{array}\right\} \Rightarrow$$

$$\Rightarrow E = \frac{20}{40} \cdot \$1 + \frac{10}{40} \cdot \$5 + \frac{5}{40} \cdot \$10 + \frac{4}{40} \cdot \$20 + \frac{1}{40} \cdot \$100$$

$$= 0.5\$ + 1.25\$ + 1.25\$ + 2\$ + 2.5\$ = 7.5\$$$

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- Today: "Measuring Risk".

Now we are gonna see how a computation of a weighted average can be applied to real-life problems concerning measuring risks.

Ex 3: (Ex 16.5.65) Suppose that you roll a pair of honest dice. If you roll a total of 7, you win \$18; if you roll a total of 11, you win \$54; if you roll any other total, you lose \$9. Find the expected payoff.

► The size of the sample set S is $N = 6^2 = 36$.
There are 6 outcomes with total sum 7, 2 outcomes with total sum 11, while the remaining $36-6-2=28$ have sum other than 7 or 11

So: $E = \frac{6}{36} \cdot \$18 + \frac{2}{36} \cdot \$54 + \frac{28}{36} \cdot (-\$9) = \$3 + \$3 - \$7 = -1\$$

Answer: the expected payoff for this game is $-1\$$ ∎

Ex 4 (Example 16.33): At the fund-raising, the raffle tickets are going for \$2. In this raffle, there is 1 grand-prize worth \$500, four second-prize winners worth \$50 each and fifteen third prize winners worth \$20 each. Suppose that 2000 raffle tickets are sold, and lets assume a raffle ticket can win only 1 prize. Find an expected gain.

► $E = \frac{1}{2000} \cdot \$500 + \frac{4}{2000} \cdot \$50 + \frac{15}{2000} \cdot \$20 + \frac{1980}{2000} \cdot \$0 = \$0.5$

E ← expected value

Hence, expected gain = expected value − ticket's value $= \$0.5 - \$2 = -\$1.5$ ∎

Def: A game is considered a fair game when no player has a built-in advantage over another player in the game.

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The last example we are gonna discuss today concerns insurances.

Ex 5 (Example 16.36): A life insurance company is offering a \$100,000 one-year term life insurance policy to Janice, a 55-year-old nonsmoking female in moderately good health. Assume that a profit margin is 20%. Find the annual premium for Janice's life insurance policy, assuming the mortality rate is 0.002 (in this group).

Let \$P be the break-even premium, i.e. the premium which would lead to a fair game, i.e. zero profit. Equivalently, it is the case of E=0.

$$\left.\begin{array}{l} E = \$(P-100{,}000)\cdot 0.002 + \$P\cdot 0.998 = \$P - \$200 \\ E=0 \end{array}\right\} \Rightarrow P=200.$$

As the profit margin is 20%, Janice should expect to pay \$200 · 1.2 = \$240 for her annual premium

Rem: Unlike gambling, in case of insurance, you better loose, that is, do not need to use your insurance at all.

* Ask if there are any questions.

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* Review Session on Mon, Dec. 12, 6^{00} - 7^{30} pm, Frey Hall 201
* Office hour on Mon, Dec. 12, 10^{00} - 11^{00}
* Final will take place on Wedn, Dec. 14, 8^{30} - 11^{00} PM in the same class as our lectures.
* Results will be available by Thur, Dec. 15, 9pm.
* You can appeal only on Fri, Dec. 16 by 6pm.