WORKSHEET 1: 12.1–12.3 VECTORS AND DOT PRODUCT

1. (a) What does the graph of $y = x^2$ look like in $\mathbb{R}^2$?
(b) What does the graph of $y = x^2$ look like in $\mathbb{R}^3$? Give a description and a picture.
(c) What does the graph of $x^2 + y^2 = 4$ look like in $\mathbb{R}^3$? Give a description and a picture.

2. Given three points $P(1, 2), Q(2, 1), R(3, 4)$, determine if the triangle ΔPQR is either a right triangle and/or an isosceles triangle.

3. Write down an equation of the sphere of radius 2 with a center at $A(1, 0, -2)$.

4. The equation $x^2 + 6x + y^2 - 2y + z^2 - 4z = 2$ determines a sphere in $\mathbb{R}^3$. Find its center and radius.

5. Find the vector represented by by the directed line segment with initial point $A(1, 0, 2)$ and terminal point $B(-1, 3, 4)$.

6. If $\overrightarrow{v} = \langle 2, -1, 1\rangle$, $\overrightarrow{w} = \langle -1, 0, 2\rangle$, find:
(a) $|\overrightarrow{v}|$.
(b) $\overrightarrow{v} - 3\overrightarrow{w}$.

7. If $\overrightarrow{v} = \langle 1, 2\rangle$, $\overrightarrow{w} = \langle 3, 1\rangle$, sketch the vectors $\overrightarrow{v} + \overrightarrow{w}$ and $\overrightarrow{v} - \overrightarrow{w}$.

8. Find a vector of length 1 in the direction opposite to $\langle 3, -4\rangle$. Express the answer in terms of the standard vectors $\overrightarrow{i}$, $\overrightarrow{j}$.

9. Compute the dot-product $\langle 1, -1, 0\rangle \cdot \langle 7, 4, 100\rangle$.

10. Are the vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 3, 1\rangle$ orthogonal? If not, determine if the angle between the vectors is obtuse or acute. Determine the angle explicitly.