HOMEWORK 1

1. (a) Let V be a nonzero finite dimensional representation of an algebra A. Show that it has an irreducible subrepresentation.

(b) Provide a counterexample to the statement of part (a) for infinite dimensional V.

2. Let A be an algebra with a unit and V be a representation of A. By $\mathrm{End}_A(V)$ we shall usually denote the algebra of all homomorphisms of A-representations $V \to V$. Prove that $\mathrm{End}_A(A) \simeq A^{\mathrm{op}}$, the algebra A with the opposite multiplication.

3. Let A be an algebra over an algebraically closed field k. The center $Z(A)$ of A is the set of all elements $z \in A$ which commute with all elements of A.

(a) Show that if V is an irreducible finite dimensional representation of A, then any element of $Z(A)$ acts in V by multiplication by some scalar $\chi_V(z)$. Show that $\chi_V \colon Z(A) \to k$ is a homomorphism. It is called the *central character* of V.

(b) Show that if V is indecomposable finite dimensional representation of A then for any $z \in Z(A)$, the operator $\rho(z)$ by which z acts in V has only one eigenvalue $\chi_V(z)$, equal to the scalar by which z acts on some irreducible subrepresentation of V. Thus $\chi_V(z) \colon Z(A) \to k$ is a homomorphism, which is again called the central character of V.

(c) Does $\rho(z)$ in (b) have to be a scalar operator?