HOMEWORK 8

1. Recall that the Specht S_n-module V_λ was realized as $V_\lambda = \mathbb{C}[S_n]a_\lambda b_\lambda$.

(a) Show that $V_\lambda \simeq \mathbb{C}[S_n]b_\lambda a_\lambda$.

(b) Show that V_λ is the image of the map from $\mathbb{C}[S_n]a_\lambda$ to $\mathbb{C}[S_n]b_\lambda$ given by right multiplication by b_λ. Likewise, show that V_λ is also the image of the map from $\mathbb{C}[S_n]b_\lambda$ to $\mathbb{C}[S_n]a_\lambda$ given by right multiplication by a_λ.

(c) Prove that $V_\lambda \otimes \mathbb{C}_- \simeq V_{\lambda'}$ as S_n-representations, where $\mathbb{C}_-$ denotes the sign representation of S_n, while λ' denotes the Young diagram conjugate to λ.

2. Use the hook length formula to characterize all irreducible representations of S_n of dimension less than n.

3. Let V be the natural n-dimensional representation of S_n. Find the multiplicities of all irreducible S_n-modules in the tensor product $V \otimes V$.