HOMEWORK 9

1. Let V be the standard n-dimensional S_n-representation (as in Problem 3 of Homework 8). For any positive integer N and any Young diagram λ of size n, find the multiplicity of the Specht module V_λ in the S_n-module $\Lambda^N V$ (the N-th exterior power of V).

2. Let V be an N-dimensional vector space, λ–a Young diagram, and $L_\lambda V$–the corresponding $\mathrm{GL}(V)$-representation. Show that $L_{\lambda+1^N} \simeq L_\lambda \otimes \Lambda^N V$ as $\mathrm{GL}(V)$-representations, where $1^N = (1, \ldots, 1) \in \mathbb{Z}^N$.

3. Let V be a 2-dimensional vector space and p, q be a pair of positive integers. Show that $S^p(S^q(V)) \simeq S^q(S^p(V))$ as $\mathrm{GL}(V)$-modules ($S^n W$ denotes the n-th symmetric power of W).