HOMEWORK 11

1. In the construction of the complementary series for $G = \mathrm{GL}_2(\mathbb{F}_q)$, we considered the representation $Y_\nu := \mathrm{Ind}_K^G \mathbb{C}_\nu$, where $K = \left\{ \begin{pmatrix} x & \epsilon y \\ y & x \end{pmatrix} : (x, y) \neq (0, 0) \right\} \subset G$ and $\mathbb{C}_\nu$ is a 1-dimensional K-representation determined by the homomorphism $\nu \colon K \to \mathbb{C}^\times$. Verify that the value of its character χ_{Y_ν} on the elliptic element $g = \begin{pmatrix} x & \epsilon y \\ y & x \end{pmatrix}$ $(x \in \mathbb{F}_q, y \in \mathbb{F}_q^\times)$ equals

$$\chi_{Y_\nu} \begin{pmatrix} x & \epsilon y \\ y & x \end{pmatrix} = \nu \begin{pmatrix} x & \epsilon y \\ y & x \end{pmatrix} + \nu^q \begin{pmatrix} x & \epsilon y \\ y & x \end{pmatrix}.$$

2. By considering the action of $\mathrm{SL}_2(\mathbb{F}_q)$ on the projective line $\mathbb{P}^1(\mathbb{F}_q)$, verify isomorphisms

$$\mathrm{SL}_2(\mathbb{F}_2) \simeq S_3, \ \mathrm{PSL}_2(\mathbb{F}_3) \simeq A_4, \ \mathrm{SL}_2(\mathbb{F}_4) \simeq A_5,$$

where A_n denotes the alternating group (a subgroup of S_n consisting of even permutations).