

## HOMEWORK 9: DETAILS FROM LECTURES 21–22

1. Verify the Key Theorem of Lecture 21 in the following two cases:
  - (a)  $\mathfrak{g} = \mathfrak{sl}_2$  and  $\lambda \in P_+$ ,
  - (b)  $\mathfrak{g}$  is arbitrary, but  $\lambda \in P_+$  is minuscule.
2. Given a simple root  $\alpha \in \Pi$ , prove that any element  $y \in U_q^-$  can be uniquely written in the form  $y = \sum_{n \geq 0} F_\alpha^{(n)} y_n$  with  $r'_\alpha(y_n) = 0$ . This is [Lecture 21, Lemma 2].
3. Prove Theorem 1 of Lecture 22 for  $\mathfrak{g} = \mathfrak{sl}_3$  (to which a general case of  $\mathfrak{g}$  was reduced).
4. Verify the formulas from Lecture 22, which relate the collection of exponents  $\mathbf{a}$  to  $\mathbf{a}'$  so that  $F_{\mathbf{i}}^{\mathbf{a}}$  coincides with  $F_{\mathbf{i}'}^{\mathbf{a}'}$  modulo  $q$  for  $\mathbf{i}'$  obtained from  $\mathbf{i}$  by a 3-term braid move.