MA 16200 Spring 2013 Lecture Topics

LECTURE 1
Chapter 12.1 Coordinate Systems
Chapter 12.2 Vectors

Let a, b, c, d be constants.

1. Describe a right hand rectangular coordinate system.
   Plot point (a, b, c) inn R^3 in a right hand rectangular coordinate system.
2. What does $x = c$ or $(y = c, \text{or } z = c)$ represent in R^2? in R^3?
3. What does the pair $x = c, y = d$ represent in R^3?
   What does the pair $y = c, z = d$ represent in R^3?
   What does the pair $z = c, x = d$ represent in R^3?
   Find distance between a point and a coordinate plane.
   Find distance between a point and a coordinate axis.
4. Find the distance between two points in R^3.
   • in a right triangle with legs of length a and b and hypotenuse of length c, $a^2 + b^2 = c^2$.
   • an isosceles triangle has 2 equal sides.
5. Find the equation of a sphere with center (a, b, c) and radius r.
   Describe the intersection of a sphere and a plane.
6. Find an equation of the sphere with center (x_1, y_1, z_1) and containing a point (x_2, y_2, z_2).
7. Complete the square(s) to write the sphere

$$x^2 + y^2 + z^2 + ax + by + cz + d = 0$$

   in standard form $((x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2)$ and find its center and radius.
8. What is the definition of a vector?
   Identify equal vectors.

LECTURE 2
Chapter 12.2 Vectors

1. Find a vector $\vec{a}$ with representation given by the directed line segment $\overrightarrow{AB}$ where $A(x_A, y_A)$ and $B(x_B, y_B)$.
   - draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.
2. Let $\vec{a}$ and $\vec{b}$ be vectors and s and t be scalars (that is, s and t are numbers). Find
   - $\vec{a} + \vec{b}$
   - $\vec{a} - \vec{b}$
   - $s\vec{a} + t\vec{b}$
   - $|\vec{a}|$
   - $|s\vec{a} + t\vec{b}|$
   - Represent vector sums and differences graphically.
3. Represent a vector using angle bracket notation and using $\vec{i}$, $\vec{j}$ and $\vec{k}$ notation.
4. Let $\vec{a}$ be a vector. Find a unit vector with the same direction as $\vec{a}$; with opposite direction.
   Let t be a positive scalar. Find a vector with the same direction as $\vec{a}$ but with length t; with opposite direction.
5. Given 2 vectors, find the direction and magnitude of the resultant vector. (The resultant vector is the sum of the 2 vectors).
6. If the two-dimentional vector $\vec{a}$ makes an angle θ with the positive x-axis, find numbers x_a and y_a such that $\vec{a} = x_a\vec{i} + y_a\vec{j}$.

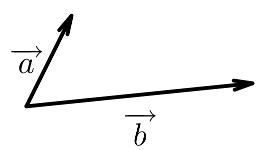
LECTURE 3

Chapter 12.3 Dot Product

1. Given $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$, find $\vec{a} \cdot \vec{b}$.
2. Given $|\vec{a}|$, $|\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$, find $\vec{a} \cdot \vec{b}$.
3. Find the angle between vectors $\vec{a}$ and $\vec{b}$.
4. Determine whether two vectors $\vec{a}$ and $\vec{b}$ are orthogonal, parallel or neither.
5. Find the direction cosines and the direction angles of a vector $\vec{a}$.
6. Find the scalar and the vector projections of $\vec{b}$ onto $\vec{a}$.
7. Find the work done by a force $\vec{F}$ acting over a displacement $\vec{D}$.
8. Determine whether the points $A(a_1, a_2, a_3)$, $B(b_1, b_2, b_3)$ and $C(c_1, c_2, c_3)$ are the vertices of a right triangle. (Don't use the pythagorean theorem).

LECTURE 4
Chapter 12.4 Cross Product

1. Find the cross product $\vec{a} \times \vec{b}$. Verify that $\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$.
2. Find $\left|\vec{a} \times \vec{b}\right|$ given $|\vec{a}|$, $\left|\vec{b}\right|$ and the angle θ between $\vec{a}$ and $\vec{b}$.
3. Find 2 unit vectors orthogonal to both $\vec{a}$ and $\vec{b}$.
4. Find the area of the parallelogram with vertices $A(x_a, y_a), B(x_b, y_b), C(x_c, y_c)$, and $D(x_d, y_d)$.
5. Given points $P(x_p, y_p, z_p), Q(x_q, y_q, z_q)$ and $R(x_r, y_r, z_r)$
   - find a nonzero vector orthogonal to the plane through the points P, Q and R.
   - find the area of the triangle PQR.
6. Find the volume of the parallelepiped determined by the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.
7. Determine if 2 vectors are parallel using their cross product.
8. Given $\vec{a} \times \vec{b}$, find $\vec{b}$ and $\vec{a}$.
9. Given the diagram below, determine whether $\vec{a} \times \vec{b}$ points into or out of the page.



LECTURE 5

Chapter 6.1 Areas between Curves

1. Find the area of a plane region R:
   - given a sketch of its boundaries and a typical approximating rectangle
   - given a sketch of its boundaries
   - given equations of its boundaries.

LECTURE 6 Volumes by Cross Section

1. Find the volume of a 3–dimensional region generated by revolving a 2–dimensional plane region about a coordinate axis:
   • given a sketch of the 2–dimensional region, the rotating axis and a typical approximating rectangular region that is rotated.
   • given the equation of the boundaries of the rotated 2–dimensional region and the axis about which it is rotated.

2. Find the volume of a 3–dimensional region generated by revolving a 2–dimensional plane region about a vertical or horizontal line that is not necessarily a coordinate axis (x = constant, or y = constant).

3. Find the volume of a 3–dimensional region given a description of its cross sections.

LECTURE 7 Volumes by Cylindrical Shells

1. Find the volume of a 3–dimensional region generated by revolving a 2–dimensional plane region about a coordinate axis:
   • given a sketch of the 2–dimensional region, the rotating axis and a typical approximating rectangular region that is rotated.
   • given the equation of the boundaries of the rotated 2–dimensional region and the axis about which it is rotated.
2. Find the volume of a 3–dimensional region generated by revolving a 2–dimensional plane region about a vertical or horizontal line that is not necessarily a coordinate axis (x = constant, or y = constant).

LECTURE 8 Chapter 6.4 Work, 6.5 Average value of a function

1. Find work done by a constant force f acting over a distance d.
2. Find work done by a variable force $f(x)$ in moving a particle along the x–axis from $x = a$ to $x = b$, $a < b$.
3. Find work done in stretching a spring from $x = a$ units beyond its natural length to $x = b$ units beyond its natural length, $a < b$.
   • given that a force s is required to hold the spring n units beyond its natural length.
   • given that t units of work are required to stretch the spring n units beyond its natural length.
4. Find work required to pump a liquid in a resevoir (tank)
   • to the top of the resevoir
   • to a level above the top of the resevoir
5. Find the average value of a continuous function, f_{ave}, over an interval $[a, b]$.
6. Find c such that $f_{ave} = f(c)$.

LECTURE 9 Chapter 7.1 Integration by Parts

1. If integrand of an integral is of the form $(u(x))\left(\frac{d}{dx}v(x)\right)$, where $u(x)$ and $\frac{d}{dx}v(x)$ are different kinds of functions, use the integration by parts formula

$$\int udv = uv - \int vdu$$

which expanded is

$$\int (u(x))\left(\frac{d}{dx}v(x)\right)dx = u(x)v(x) - \int (v(x))\left(\frac{d}{dx}u(x)\right)dx$$

The integral on the left of the equal sign is the one you want to evaluate. Instead, evaluate the right hand side. What you do is
1. Determine u and dv (Let u be the left–most function in the list: Log, Inverse trig, Algebraic, Trig, Exponential, LIATE)
2. Let dv be the other factor in the integrand (together with dx)
3. Compute du from u and v from dv.
4. Evaluate $uv - \int vdu$.

LECTURE 10 Chapter 7.2 Trigonometric Integrals

1. Recognize integrals of derivatives of trig functions, for example, $\int \sec^2 x \; dx = \tan x + C$.

2. Rewrite integral in form $\int u^n \; du$ (or a linear combination of these integrals) by using the trig identitity $\sin^2 x + \cos^2 x = 1$ and
   • factoring $\sin x \; dx$ leaving powers of $\cos x$ or
   • factoring $\cos x \; dx$ leaving powers of $\sin x$.

3. Rewrite integral in form $\int u^n \; du$ (or a linear combination of these integrals) by using the trig identitity $\tan^2 x + 1 = \sec^2 x$ and
   • factoring $\sec^2 x \; dx$ leaving powers of $\tan x$ or
   • factoring $\tan x \sec x \; dx$ leaving powers of $\sec x$.

4. Rewrite integral in form $\int u^n \; du$ (or a linear combination of these integrals) by using the trig identitity $\sec^1 x - 1 = \tan^2 x$ and
   • factoring $\csc^2 x \; dx$ leaving powers of $\csc x$ or
   • factoring $\csc x \cot x \; dx$ leaving powers of $\cot x$.

5. For even powers (only) of $\sin x$ and $\cos x$, use the following trig identities:
   • $\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$.
   • $\cos^2 x = \frac{1}{2} + \frac{\cos 2x}{2}$.

6. Another useful trig identity is $\sin 2x = 2 \sin x \cos x$.

7. Sometimes, when all else fails, change everything to sin's and cos's.

LECTURES 11 and 12 Chapter 7.3 Trig Subsitution

If integrand contains*	Use this substitution	And use this identity
$\sqrt{a^2 - (u(x))^2}$	$u(x) = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + (u(x))^2}$	$u(x) = a\tan\theta$	$1 + \sin^2\theta = \sec^2\theta$
$\sqrt{(u(x))^2 - a^2}$	$u(x) = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

* Sometimes the integrand will contain powers of these roots.

Completing the square is sometimes necessary to get the forms $a^2-(u(x))^2,\ a^2+(u(x))^2,$ or $(u(x))^2-a^2$.

LECTURE 13 Chapter 7.4 Partial Fractions

1. Write out the form of the partial fraction decomposition of a rational function. Remember that the degree of the numerator must be less than the degree of the denominator (if not, divide) and that numerator and denominator cannot have common factors (if not, cancel the common factors).
2. Evaluate integrals of rational functions where the denominator factors into linear factors only.

LECTURE 14 Chapter 7.4 Partial Fractions (continued)

1. Evaluate integrals of rational functions where the denominator factors into linear and/or irreducible quadratic factors.
2. Evaluate integrals where integrand contains roots of linear expressions, for example $\sqrt{x+2}$. First use a u substitution, for example $u = \sqrt{x+2}$. Then use partial fractions as needed.

LECTURE 15 Chapter 7.6 Integration Using Tables, 7.7 Approximate Integration

1. Evaluate an integral using a table of integrals. Be careful when substituting for dx.

2. Find the approximate value of the definite integral $\int_a^b f(x)dx$ using the Midpoint Rule, the Trapeziod Rule and Simpson's Rule with n subintervals.

LECTURE 16 Chapter 7.8 Improper Integrals

1. Evaluate improper integrals of the form $\int_a^\infty f(x)dx$ and $\int_{-\infty}^b f(x)dx$ where f is a continuous function.
2. Evaluate an improper integral of the form $\int_a^b f(x)dx$ where f has an infinite discontinuity at $x = c$, where $a \le c \le b$.
3. Evaluate an improper integral of the form $\int_{-\infty}^\infty f(x)dx$.

LECTURE 17 Chapter 8.1 Arc Length, 8.2 Area of Surface of Revolution

1. Find the length of the curve $y = f(x)$, $a \leq x \leq b$.
2. Find the length of the curve $x = g(y)$, $c \leq y \leq d$.
3. Find the area of the surface of revolution obtained by revolving the curve $y = f(x)$, $a \leq x \leq b$ about the x–axis.
4. Find the area of the surface of revolution obtained by revolving the curve $y = f(x)$, $a \leq x \leq b$ about the y–axis.
5. Find the area of the surface of revolution obtained by revolving the curve $x = g(y)$, $c \leq y \leq d$ about the x–axis.
6. Find the area of the surface of revolution obtained by revolving the curve $x = g(y)$, $c \leq y \leq d$ about the y–axis.

LECTURE 18 Chapter 8.3 Centroids and center of mass

1. Find the moment about the x–axis (M_x) and the moment about the y–axis (M_y) of a lamina.
2. Find the centroid $(\overline{x}, \overline{y})$ of a lamina.

LECTURE 19 Chapter 11.1 Sequences

1. List the first few terms of a sequence $\{a_n\}$.
2. Determine whether a sequence $\{a_n\}$ converges or diverges. If it converges, find its limit.
3. Determine whether a sequence $\{a_n\}$ is increasing, decreasing or not monotonic.
4. Determine whether a sequence $\{a_n\}$ is bounded.

LECTURE 20 Infinite Series

1. Find the first few partial sums of an infinite series.
2. Use the "Great Divergence Test" to determine whether an infinite series diverges.

   If $\lim a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.
3. Determine whether a geometric series converges or diverges. If it converges, find its sum.

LECTURE 21 Chapter 11.3 The Integral Test and p–series

1. Use the integral test, where appropriate, to determine whether a series converges or diverges.
2. Determine whether a p–series series converges or diverges.

LECTURE 22 Chapter 11.4 Comparison Tests

1. Use the Comparison Test, where appropriate, to determine whether a series converges or diverges.
2. Use the Limit Comparison Test, where appropriate, to determine whether a series converges or diverges.

LECTURE 23 Chapter 11.5 Alternating Series

1. Determine whether an alternating series converges using the Alternating Series Test.
2. Determine the smallest N such that the Nth partial sum, S_N, approximates the value of the series to within a given accuracy.

LECTURE 24 Chapter 11.6 Absolute convergence, conditional convergence and the Ratio Test.

1. Determine whether a series is absolutely convergent, conditionally convergent or divergent.
2. Use the Ratio Test to determine whether a series is convergent.

LECTURE 25 Chapter 11.6 Root test and 11.7 Strategy for Series

1. Use the Root Test to determine whether a series is convergent.
2. Determine whether a series is convergent or divergent, using any an appropriate convergence test studied so far.

LECTURE 26 Chapter 11.8 Power Series

1. Determine the interval of convergence and the radius of convergence for the power series $\sum_{n=0}^{\infty} c_n (x-a)^n$.

LECTURE 27 Chapter 11.9 Power Series Representation of a function

1. Find the power series representation for a function $f(x)$ based on the geometric series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \ -1 < x < 1$$

using substitution of a constant or a variable expression for x, multiplication of the series by a constant or a variable expression, differentiation, and/or integation.

LECTURE 28 Chapter 11.10 Taylor and Maclaurin Series

1. Find the Taylor series for $f(x)$ at $x = a$ using the definition.
2. Find the Maclaurin series for $f(x)$ using the definition.

LECTURE 29 Chapter 11.10 Taylor and Maclaurin Series

1. Find the Taylor series for $f(x)$ at $x = a$ using substitution, multiplication by constants and variable expressions, differentiation and integration.
2. Find the Maclaurin series for $f(x)$ using substitution, multiplication by constants and variable expressions, differentiation and integration.
3. Evaluate indeterminate limits (0/0) using power series representations rather than l'Hopital's Rule.